1. Prove that PCA minimizes $E_{x,y}[d(x,y)^2 - d(x',y')^2]$ where $d$ is the Euclidean distance function, the $x$ and $y$ are original data samples, and $x'$ and $y'$ are data samples after PCA projection.

2. For the matched filter considered in Haykin, Example 8.2, the eigenvalue $\lambda_1$ and associated eigenvector $q_1$ are defined by

$$\lambda_1 = 1 + \sigma^2$$
$$q_1 = s$$

Show that these parameters satisfy the basic relation

$$Rq_1 = \lambda_1 q_1$$

where $R$ is the correlation matrix of the input vector $X$.

3. Consider the maximum eigenfilter where the weight vector $w(n)$ evolves in accordance with Haykin, Eq. (8.46). Show that the variance of the filter output approaches $\lambda_{max}$ as $n$ approaches infinity, where $\lambda_{max}$ is the largest eigenvalue of the correlation matrix of the input vector.

4. Show that in Kernel PCA, the normalization of eigenvector $\tilde{q}$ of the correlation matrix $\tilde{R}$ is equivalent to the requirement that Haykin, Eq. (8.153) be satisfied.