Tik-61.140 Signal Processing Systems

2nd midterm, Mon 3.5.1999 16-19, hall ABC

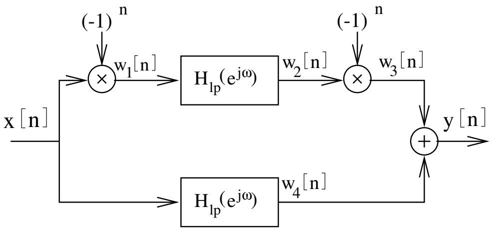
1. Determine whether the propositions are true.
   (a) Signal multiplication in time domain corresponds convolution of Fourier transforms in frequency domain.
   (b) Discrete Fourier transform is linear.
   (c) Discrete lowpass filter (cutoff frequency $\pi/4$) and high pass filter (cutoff frequency $3\pi/4$) parallelly connected form a bandpass filter.
   (d) The convolution of an odd Fourier-transform $X_1(j\omega)$ with an even Fourier-transform $X_2(j\omega)$ is always odd.
   (e) The Fourier series coefficents of $sin(\omega_0 t)$ are a_1 = -1/2, a_1 = 1/2 and a_k=0 for other k.
   (f) Ideal discrete lowpass filter is causal.

2. Consider a causal discrete-time LTI system determined by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1]$$

   where $|a|<1$ and a is real.
   (a) Determine b, so that the amplitude response $|H(e^{j\omega})| = 1$ for all ω. Note that the constant b must not depend on ω.
   (b) Compute the response $y[n]$ for the input $x[n] = (\frac{1}{3})^n u[n]$, when $a = -\frac{1}{3}$ and b as in (a). Note that it is not enough to list values of $y[n]$, the solution must be given in a closed form expression for $y[n]$.

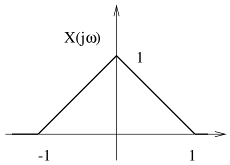
3. Let us consider a discrete system with input $x[n]$ and output $y[n]$ in the figure below. LTI-systems $H_{lp}(e^{j\omega})$ are ideal lowpass filters with cutoff frequency $\pi/6$ and unity gain in the passband. Find the frequency response of the total system using signals w and properties of discrete Fourier transform. What frequency properties does the system have? Hint: $(-1)^n = e^{j\pi n}$.



4. Let us consider sampling from a continuous-time signal $x(t)$. The samples are obtained by multiplying the signal with a sampling function which has a Fourier-series representation of

$$p(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{j(2\pi nt/T)},$$

   that is, $x_p(t) = x(t)p(t)$. Here T is the period of the sampling function. Calculate the Fourier transform $X_p(j\omega)$ of $x_p(t)$, assuming that $X(j\omega)$ is known.
   Consider a situation where $X(j\omega)$ is as shown in the figure below



   Plot the magnitude of the Fourier transform $X_p(j\omega)$ if the sampling frequency $\omega_s = 2\pi/T$ is
   (a) $\omega_s = 2$
   (b) $\omega_s = 3/2$.