T-61.140 Signal Processing Systems

2nd mid term exam, Fri 3.5.2002 12-15 M (Simula, Parviainen)
You may use a graphical calculator of your own. There are formulae on accompanying papers - use them!

1) (0-3p) Answer, if the statement is true (T) or false (F). Correct answer +1p, wrong -1p, no answer 0p. Explain briefly!
ATT! There are four statements, max points 3.

   a) Convolution of input signal and impulse response in time-domain corresponds the sum of frequency response and Fourier-transform of input signal.

   b) Given amplitude spectrum $|X(e^{j\omega})|$ of any signal $x[n]$, it is possible to recover time-domain sequence $x[n]$ from amplitude spectrum, if there is enough calculation power in the computer.

   c) If $h_{lp}[n]$ is impulse response of a low pass filter, then a high pass filter $h_{hp}[n]$ can be obtained by multiplying $h_{lp}[n]$ with $(-1)^n$.

   d) Continuous-time Fourier-transform is periodic with the period of 2π.

2) (3p) Answer with a few sentences.

   a) What do terms analog(ue), discrete and digital signal mean?

   b) Why is it useful to process signal digitally?

3) (6p) LTI system is defined with a difference equation

$$y[n] = 0.25\,x[n] - 0.5\,x[n-1] + 0.25\,x[n-2]$$

   a) (1p) Draw the block (flow) diagram of LTI system.

   b) (2p) What is the frequency response of $H(e^{j\omega})$?

   c) (1p) Sketch $|H(e^{j\omega})|$ in range $0..\pi$. Is the system of type lowpass / highpass / bandpass / bandstop?

   d) (2p) Compute the output $y[n]$, when the input is $x[n] = \{0,\ 1,\ 1,\ 2,\ 1,\ 8,\ 2,\ 1,\ 1\}$. Draw both $x[n]$ and the filtered $y[n]$.

4) (6p) Consider a continuous-time signal $x(t)$, which consists of three cosine components (150 Hz, 350 Hz, 450 Hz)

$$x(t) = \cos(2\pi\ 150\,t) + \cos(2\pi\ 350\,t) + \cos(2\pi\ 450\,t)$$

   a) (2p) Sketch the spectrum $|X(j\omega)|$ in range 0..1500 Hz.

   b) (1p) What is the lowest sampling frequency, with which there is no aliasing?

   c) (3p) Demonstrate the effect of too low sampling frequency in the spectrum of discrete-time signal $|X(e^{j\omega})|$. Choose the sampling frequency between $350 < f_s < 450$.

Turn!

5) (6p) **Reply to either A or B.**

5A) The calculator of the assistant is broken. You should help him and produce a sequence of squares {..., 0, 1, 4, 9, 16, 25, ...} using a third order recursive LTI filter (allowed operations: sum of signals, amplification of constants, delay). Figure out or compute the coefficients of the filter and determine the initial values of delay registers.

Hints:

- Examine $y[n] = n^2, \; y[n-k] = (n-k)^2$
- Examine sequences $\{0, 1, 4, 9, 16, 25, ...\}$,
$\{(1-0), (4-1), (9-4), (16-9), (25-16), ...\}$,
$\{(4-1)-(1-0), (9-4)-(4-1), (16-9)-(9-4), (25-16)-(16-9), ...\}$
- Keep the input always as zero ($x[n] \equiv 0$, for all n).

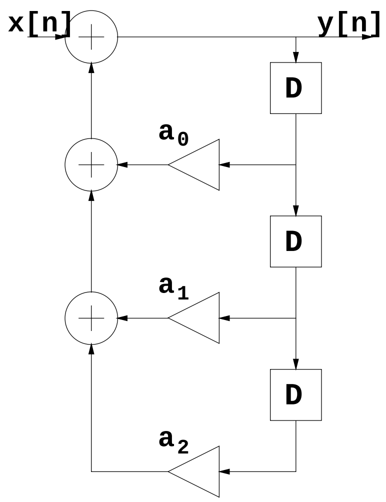

Figure 1: Recursive third order IIR filter (without FIR part).

5B) In the study program 2001-2002 there is a keyword "frequency analysis of signals" in the contents of T-61.140 Signal Processing Systems. Write down an essay on this subject. The maximum length of the essay is two pages with line space of two; illustrations can be drawn to clarify the text.