BLIND SEPARATION OF NOISY HARMONIC SIGNALS USING ORTHOGONAL TECHNIQUES FOR ROTATING MACHINE DIAGNOSIS

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ABSTRACT

We present a robust-to-noise technique for the blind source separation of harmonic signals, using temporal correlations. The mixtures are convolutive and corrupted with spatially correlated noises. Their distributions and correlation functions are unknown. As the usual Principal Components Analysis cannot provide a correct estimate of the signal subspace in this situation, we introduce a new estimator of the decorrelation matrix and the signal subspace. It exploits the model of harmonic sources and uses non-hermitian interspectral matrices of delayed observations. After whitening, the delayed observations are used again to determine the Givens plane rotations that complete the separation process. A Wiener filtering is then applied for denoising the outputs. The proposed method is efficient for low Signal to Noise Ratio and illustrated with simulations and rotating machine vibration data.

1. INTRODUCTION

The Blind Source Separation has largely been investigated in many fields (noise reduction, radar and sonar processing, speech enhancement, separation of rotating machine noises, localization in array processing...). It is a promising technique for rotating machine monitoring by vibrations analysis. It consists in recovering the specific signature of each rotating system (called source) from a set of sensors [1], [8]. The observations are assumed to be convolutive mixtures of the harmonic sources, corrupted with noise or other mechanical systems. Solutions have been proposed, which generally suppose that the observations are unnoisy or corrupted with spatially white, or gaussian processes [6], [5]. This paper addresses the issue of orthogonal techniques for Blind Source Separation of periodic signals in presence of spatially correlated noises. This problem is of major interest for experimental signals processing.

Let us consider a linear and stationary system with p inputs (sources) and n ($n \geq p$) outputs (sensors). The observations are convolutive mixtures of the harmonic sources, corrupted with additive noises that may be spatially and temporally correlated. Both the sources and the noises are assumed to be zero mean. The sources are assumed to be independent but they may have closely spaced frequencies. The operations are processed in the frequency domain, where the mixtures are instantaneous at every frequency bin. The Discrete Fourier Transform (DFT) of the observation vector $\mathbf{r}(t)$ on the temporal window $[t, \ldots, t+N-1]$ is denoted $\mathbf{r}_N^t(f)$. These observations are modeled by

$$\mathbf{r}_N^t(f) = \underbrace{\mathbf{H}(f) \cdot \mathbf{s}_N^t(f)}_{\mathbf{y}_N^t(f)} + \mathbf{b}_N^t(f) \; ,$$

- $\mathbf{r}_N^t(f)$ is the $(n \times 1)$ observation vector,
- $\mathbf{H}(f)$ is the $(n \times p)$ mixing matrix including the source power spectral densities,
- $\mathbf{s}_N^t(f)$ is the $(p \times 1)$ vector of the normalised sources, $\mathbf{b}_N^t(f)$ is the $(n \times 1)$ noise vector ; the noises are assumed to be decorrelated from the sources.

To simplify the notations (f) will be omitted. The separation issue consists first in determining the pseudo-inverse of $\mathbf{H}$. The Singular Value Decomposition (SVD) of this matrix may be written $\mathbf{H} = \mathbf{V}\Delta^{1/2}\Pi$ where $\mathbf{V}$ and Π are unitarian matrices and $\Delta^{1/2}$ (for convenience) is the diagonal matrix of singular values. This matrix can be inversed in two steps. The first step aims to decorrelate the mixtures as follows :

$$\mathbf{x}_N^t = \mathbf{W} \cdot \mathbf{r}_N^t \quad \text{with} \quad \mathbf{W} = \Delta^{-1/2}\mathbf{V}^+ . \qquad (1)$$

The whitening matrix $\mathbf{W}$ is obtained from the Eigen Value decomposition (EVD) of the interspectral matrix of noiseless mixtures :

$$\gamma_{\mathbf{y}} = E\left\{\frac{\mathbf{y}_N^t \cdot \mathbf{y}_N^{t\,+}}{N}\right\} = \mathbf{H}\mathbf{H}^+ = \mathbf{V}\Delta\mathbf{V}^+ . \qquad (2)$$

The main problem is then to estimate $\gamma_{\mathbf{y}}$ from the noisy observations. A Principal Components Analysis (PCA) is usually performed but this method is not robust as it assumes strong hypothesis on the noises (gaussianity and spatial whiteness). The eigenvectors and eigenvalues of $\gamma_{\mathbf{y}}$ are ill estimated and the PCA looses efficiency. The estimation of the source number can fail and the whitening matrix is not correctly estimated because of errors on the signal subspace dimension and on estimated eigenvalues and eigenvectors matrices $\Delta^{1/2}$ and $\mathbf{V}$. We exploit here the model of harmonic sources to introduce a new estimator of the decorrelation matrix $\mathbf{W}$ and of the signal subspace. It uses non-hermitian interspectral matrices of delayed observations to build "noiseless" interspectral matrices. Two of these non-hermitian matrices provide a direct estimator of the unnoisy spectral matrix $\gamma_{\mathbf{y}}$, and therefore an estimator of the whitening matrix. These aspects are developed in section 2. Simulations results in section 3 show the efficiency of the new estimator and measure jointly the accuracy of the estimated eigenvalues and the closeness to the signal subspace.

After projection of the observations on the signal subspace and normalization, the second step of the Blind Source Separation process consists in the estimation of matrix Π. In the noiseless case or in the case of gaussian noises, Π is normally obtained by the use of higher order statistics of $\mathbf{x}_N^t$ [6]. In the non gaussian case, or in the case of very low Signal to Noise Ratios (SNR), such a method generates important estimation errors. We show in section 4, that the EVD of any delayed spectral matrix of $\mathbf{x}_N^t$ provides matrix Π, as previously proved in the case of instantaneous mixtures [2]. Concluding this stage, every output contains one source component and some noise. The last step, described in section 5, consists in denoising the outputs using a Wiener filter to maximize the SNR relative to each component of the source vector. Finally, a high noise level simulation and experimental results on machine vibration data are given for the whole Blind Source Separation procedure in sections 6 and 7 .

2. A NEW ESTIMATOR OF THE SIGNAL SUBSPACE

We define the delayed interspectral matrix of observations by the following :

$$\gamma_{\mathbf{rr}}^{\tau} = E\left\{\frac{\mathbf{r}_N^t \cdot \left(\mathbf{r}_N^{t+\tau}\right)^+}{N}\right\} = \gamma_{\mathbf{yy}}^{\tau} + \gamma_{\mathbf{bb}}^{\tau} ,$$

where $^+$ stands for the transpose-conjugate, $\mathbf{r}_N^t$ and $\mathbf{r}_N^{t+\tau}$ are the N points DFTs of the temporal observation vectors $\mathbf{r}(t)$ and $\mathbf{r}(t+\tau)$. In the case of periodic sources, it is interesting to exploit the fact that the autocorrelation lengths of the sources are larger than the correlation lengths of all the noises. Denote τ_B the largest correlation or cross-correlation length of the noises. Since the sources have infinite correlation lengths, if $N + \tau_B \leq \tau$, then $\mathbf{r}_N^t$ and $\mathbf{r}_N^{t+\tau}$ contain contributions from decorrelated noise variables and correlated source variables. The consequence is that $\gamma_{\mathbf{bb}}^{\tau} = \mathbf{0}$ and

$$\gamma_{\mathbf{rr}}^{\tau} = \gamma_{\mathbf{yy}}^{\tau} = \mathbf{H}\gamma_{\mathbf{ss}}^{\tau}\mathbf{H}^+ . \tag{3}$$

The rank of $\gamma_{\mathbf{yy}}^{\tau}$, denoted c, corresponds to the number of energetic sources at the frequency bin f. The sources are independent and normalised, so $\gamma_{\mathbf{ss}}^{\tau}$ is a diagonal matrix in the form

$$\gamma_{\mathbf{ss}}^{\tau} = \begin{pmatrix} e^{-j2\pi f_1\tau} & & 0 \\ & \ddots & \\ 0 & & e^{-j2\pi f_c\tau} \end{pmatrix} = \theta .$$

The f_is are the frequencies of the energetic sources close to the frequency bin of analysis f.

Introducing the SVD of $\mathbf{H}$ in (3) we obtain

$$\gamma_{\mathbf{rr}}^{\tau} = \mathbf{V}\Delta^{1/2}\Pi\theta\Pi^+\Delta^{1/2+}\mathbf{V}^+.$$

With this decomposition, it is easy to show that another interspectral matrix, with delay -2τ, allows for recovering $\gamma_{\mathbf{yy}}$ according to

$$\gamma_{\mathbf{yy}} = \gamma_{\mathbf{rr}}^{\tau} \cdot \left(\gamma_{\mathbf{rr}}^{\tau+}\right)^{-1} \cdot \gamma_{\mathbf{rr}}^{-2\tau} .$$

This expression involves the inversion of a matrix of size n but rank c. As we assume to know the number of sources c, stability is gained using the pseudo-inverse algorithm. Finally, the EVD of $\gamma_{\mathbf{yy}}$ gives $\Delta^{1/2}$ and $\mathbf{V}$

and the matrix filtering (1) may be performed. When the source number is unknown, a robust-to-noise estimation can be performed using the delayed interspectral matrix of observations as shown in [7].

3. DISTANCE TO THE SIGNAL SUBSPACE

As we said in section 2, the efficiency of PCA relies on the estimation of the source number and the estimation of an orthonormal base of $\mathbb{E}_{\mathbf{S}}$. We now need a criterion to measure jointly the accuracy of the estimated eigenvalues and the closeness to the signal subspace. Denote $\|.\|$ the matrix 2-norm, and $\widehat{\gamma_{yy}}$ the estimate of $\gamma_{\mathbf{yy}}$. The distance $\|\gamma_{yy} - \widehat{\gamma_{yy}}\|$ is not appropriate since $\widehat{\gamma_{yy}}$ can be close to γ_{yy} without the good eigenvalues and eigenvectors. Consequently the criterion must rely on the whitening matrix $\mathbf{W}$. The usual rejection rates referred in [4] cannot be used here since the rotation matrix Π is undetermined after the PCA. We must pay attention to the fact that the estimated whitening matrix $\widehat{\mathbf{W}}$ is not uniquely determined: it can be left multiplied by a unitarian matrix [5]. Denote $\widehat{\mathbf{W}} = \tilde{\mathbf{D_s}}^{-1/2}.\tilde{\mathbf{V_s}}^{+}$. If the column vectors in $\tilde{\mathbf{V_s}}$ form an orthonormal base of $\mathbb{E}_\mathbf{S}$ then $\tilde{\mathbf{V_s}}^{+}.\mathbf{V_s}$ is close to a unitarian matrix $\mathbf{P}$. When the mixing matrix $\mathbf{H}$ is unitarian, $\mathbf{P}$ is diagonal. In this situation, when the estimated eigenvalues are close to the real ones, the norm of (4) is close to 1 (norm of $\mathbf{P}$). # denotes the pseudo-inverse. This considerations leads to the distance criterion (5):

$$\widehat{\mathbf{W}}.\mathbf{W}^{\#} = \tilde{\mathbf{D_s}}^{-1/2}.\tilde{\mathbf{V_s}}^{+}.\mathbf{V_s}.\mathbf{D_s}^{1/2} \tag{4}$$

$$d(\widehat{\mathbf{W}}, \mathbf{W}) = \left| \quad \|\widehat{\mathbf{W}}.\mathbf{W}^{\#}\| - 1 \quad \right| \tag{5}$$

We show simulation results on figure 1, 2 and 3. Two sources are mixed and observed on 6 sensors. Each source is composed of 2 pure frequencies (0.14,0.36 and 0.15,0.37). The mixture is obtained from AR1 filters with coefficients in the range $[0, 1]$ so that some filters are low-pass and others are high-pass. The spatially correlated and spectrally colorated noises result from the filtering of white noises with AR1 filters too. As shown on the table below the noise power spectral densities on the frequency bin 0.15 are different on every sensor and the corresponding Signal to Noise Ratios are about -5 dB.

sensors	Noise PSD	SNR in dB
1	0.17	-5.01
2	0.32	-5.03
3	0.14	-5.01
4	0.21	-5.02
5	0.43	-5.00
6	0.09	-5.00

Computations are processed on 612 sliding blocks of 64 samples with $\tau = 70$ samples. The sliding step is 16 samples.

Denote γ_1 and γ_2, the estimation of γ_{yy} respectively with the usual PCA method, and the new method involving the delay τ. $d1$ and $d2$ are the corresponding distances to the signal subspace. In this simulation we assume that the number of sources is known, because we just study here the signal subspace estimation. In figure 1 one can see that the eigenvalues of γ_1 in dotted line are very far from the eigenvalues of γ_{yy} in solid line whereas in figure 2, the eigenvalues of γ_2 are very close to the eigenvalues of γ_{yy}. The distance $d2$ to the signal subspace is much lower than the distance $d1$ as shown in figure 3.

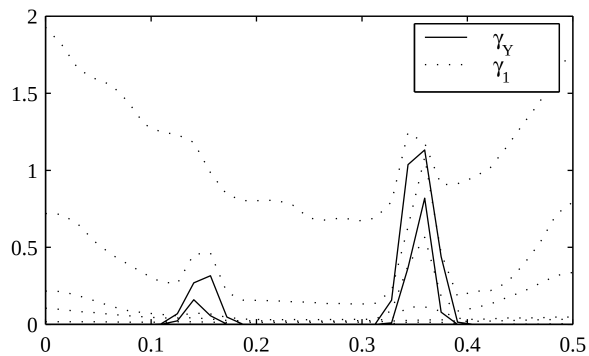

Figure 1: eigen values of γ_{yy} and γ_1

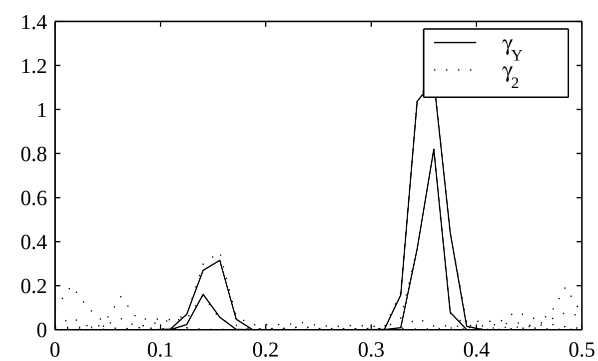

Figure 2: eigen values of γ_{yy} and γ_2

4. ORTHOGONAL TRANSFORMATION STEP

After estimation of the signal subspace and the whitening matrix $\mathbf{W}$, as detailed in parts 2 and 3, the observations are projected on an orthonormal base of the signal subspace and normalized.

The decorrelating of $\mathbf{r}_N^t$ and $\mathbf{r}_N^{t+\tau}$ provides vectors

$$\mathbf{x}_N^t = \Pi \cdot \mathbf{s}_N^t + \mathbf{W} \cdot \mathbf{b}_N^t$$

and

$$\mathbf{x}_N^{t+\tau} = \Pi \cdot \mathbf{s}_N^{t+\tau} + \mathbf{W} \cdot \mathbf{b}_N^{t+\tau}.$$

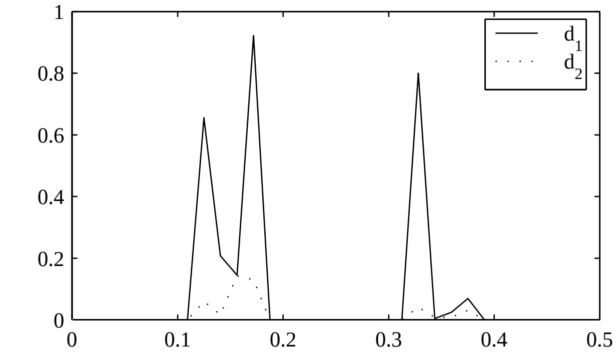

Figure 3: distance to $\mathbb{E}_S$ estimated from γ_1 and γ_2

Remark that in the case of spatially correlated noises, the noise components are still correlated. The interspectral matrix constructed from these vectors is free of noise correlation :

$$\gamma_{\mathbf{xx}}^{\tau} = E\left\{\frac{\mathbf{x}_N^t \cdot \left(\mathbf{x}_N^{t+\tau}\right)^+}{N}\right\} = \Pi\theta\Pi^+$$

Π is the eigenvector matrix of $\gamma_{\mathbf{xx}}^{\tau}$ as long as θ is different of the identity matrix. We also remark that the eigenvalues of $\gamma_{\mathbf{xx}}^{\tau}$ are estimators of the f_is, which may be useful for rotating machine monitoring. We may thereby conclude the pseudo-inversion of $\mathbf{H}$ with $\mathbf{H}^{\#} = \Pi^+\mathbf{W}$ and complete the separation process following :

$$\mathbf{z}_N^t = \Pi^+ \cdot \mathbf{x}_N^t = \mathbf{s}_N^t + \mathbf{H}^{\#} \cdot \mathbf{b}_N^t \quad . \qquad (6)$$

Notice that the EVD of any delayed spectral matrix $\gamma_{\mathbf{xx}}^{\tau}$ provides the estimate of Π to the extent of a complex permutation matrix $\mathbf{P}$:

$$\widehat{\Pi} = \Pi\mathbf{P}.$$

Performance of the method is investigated in part 6. Its robustness can be improved by joint diagonalization of a set of these delayed spectral matrices as in [2], if all the values τ verify $N + \tau_B \leq \tau$. Nevertheless, no estimator of the f_is is available in that case. Concluding this stage, every output contains one source component and still some spatially correlated noises.

5. NOISE REDUCTION STEP

The SNR relative to each component of $\mathbf{z}_N^t$ depends on the coefficients of $\mathbf{H}^{\#}$ and on the correlation coefficients of the noise components in vector $\mathbf{b}_N^t$. It is possible to maximize the SNR relative to the ith source, through a multi-reference Wiener filtering in which the observation is the ith component of $\mathbf{z}_N^t$ and the references are the $(c-1)$ remaining.

Let's simplify the notations using (7) instead of (6)

$$\mathbf{z} = \mathbf{s} + \mathbf{v} \qquad (7)$$

- $\mathbf{s}^T = [s_1, \ldots, s_c]$: DFTs of the sources,
- $\mathbf{v}^T = [v_1, \ldots, v_c]$: DFTs of the correlated noises.

The noise reduction operation consists in estimating and substracting the noise components of $\mathbf{z}$ in order to get a better estimate of the source :

$$\widetilde{s}_i = z_i - \widehat{v}_i, \quad i = 1 \ldots c.$$

The ith noise component of $\mathbf{z}$ is estimated by a linear combination of the $(c-1)$ remaining components of $\mathbf{z}$:

$$\widehat{v}_i = \sum_{k \neq i} F_k \cdot z_k = \mathbf{z}_{-i}^T \cdot \mathbf{F}$$

with

$$\mathbf{z}_{-i}^T = [z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_c]$$
$$\mathbf{F}^T = [F_1, \ldots, F_{i-1}, F_{i+1}, \ldots, F_c] \ .$$

The Wiener coefficients in $\mathbf{F}$ minimize the Mean Square Error $\varepsilon^2 = \mathrm{E}\left[|v_i - \widehat{v}_i|^2\right]$. The solution is obtained from the orthogonal projection theorem :

$$\mathbf{F} = \left(\mathbf{R}_{\mathbf{z}_{-i}\mathbf{z}_{-i}}^*\right)^{-1} \cdot \mathbf{R}_{z_i\mathbf{z}_{-i}} \ ,$$

with

$$\mathbf{R}_{\mathbf{z}_{-i}\mathbf{z}_{-i}} = \mathrm{E}\left[\mathbf{z}_{-i} \cdot \mathbf{z}_{-i}^+\right]$$
$$\mathbf{R}_{z_i\mathbf{z}_{-i}} = \mathrm{E}\left[z_i \cdot \mathbf{z}_{-i}^*\right].$$

and the SNR gain is

$$G_i = \frac{SNR_{out}}{SNR_{in}} = \frac{1}{1 - \dfrac{\mathbf{R}_{z_i\mathbf{z}_{-i}}^+ \cdot \left(\mathbf{R}_{\mathbf{z}_{-i}\mathbf{z}_{-i}}^*\right)^{-1} \cdot \mathbf{R}_{z_i\mathbf{z}_{-i}}}{R_{v_iv_i}}}$$

The sources are recovered and denoised up to a permutation indetermination. Some techniques exist to remove it, for example see [3]

6. SIMULATION RESULTS

Three harmonic sources with closely spaced frequencies ($f_1 = 0.1, f_2 = 0.11, f_3 = 0.12$) are filtered through fifteen AR1 filters and received on five sensors. The five observations are corrupted by temporally white and spatially correlated noises resulting from the filtering of a gaussian and a uniformly distributed noises (ten AR1 filters). The observations are processed at a frequency bin $f = 0.11$ and the SNR is 0dB. The interspectral matrices are estimated on 900 DFT blocks of 64 samples with 32 sample overlapping. The delay τ is equal to 72 samples. The source number is known.

The accuracy of estimation of the decorrelation matrix may be measured with the relative errors on the components of $\widehat{\mathbf{W}}$. These terms are gathered in the $c \times n$ matrix $\mathbf{E}$ below :

$$\mathbf{E} = (E_{ij}) \quad \text{with} \quad E_{ij} = \left| \frac{\widehat{W_{ij}} - W_{ij}}{W_{ij}} \right|$$

The relative error matrix is

$$\mathbf{E} = \begin{pmatrix} 0.027 & 0.017 & 0.080 & 0.015 & 0.412 \\ 0.021 & 0.064 & 0.049 & 0.022 & 0.035 \\ 0.056 & 0.024 & 0.063 & 0.097 & 0.039 \end{pmatrix}$$

All but one relative error of the elements in $\widehat{\mathbf{W}}$ are much lower than 10% which is reasonable in this very noisy situation.

In order to measure the accuracy of estimation of Π the observations are decorrelated using $\mathbf{W}$ (instead of $\widehat{\mathbf{W}}$). Let's consider matrix $\mathbf{Q} = \Pi^{+}\widehat{\Pi}$. When the estimation is perfect $\mathbf{Q}$ is a permutation matrix (see section 4). The absolute elements of $\mathbf{Q}$ are the following:

$$(|Q_{ij}|) = \begin{pmatrix} 0.015 & 1.002 & 0.028 \\ 0.028 & 0.013 & 1.067 \\ 0.991 & 0.014 & 0.020 \end{pmatrix}$$

We can conclude that the estimation of $\widehat{\Pi}$ is very accurate. Notice that the first output corresponds to the second source, the second output to the third source and third output to the first source. The figures below represent successively the envelopes of a noisy observation, a noisy separated output (using $\widehat{\mathbf{W}}$ and $\widehat{\Pi}$), a denoised output and one normalized source. The SNR are respectively 0dB, 6dB and 25dB. The noise reduction step is very efficient since we have here two references to denoise each output.

7. EXPERIMENTAL RESULTS ON MACHINE VIBRATION DATA

The results here after illustrate a complete implementation of the algorithm (separation step and Wiener filtering) on a real process. It involves two test beds. Each one includes a synchronous alternator, a motor and a pump. The alternator rotation speeds are 50Hz and 23Hz. The two test beds are mechanically linked

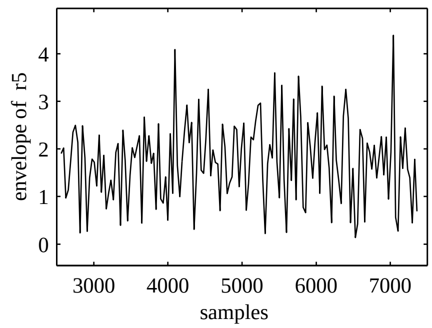

Figure 4: Envelope of a noisy observation

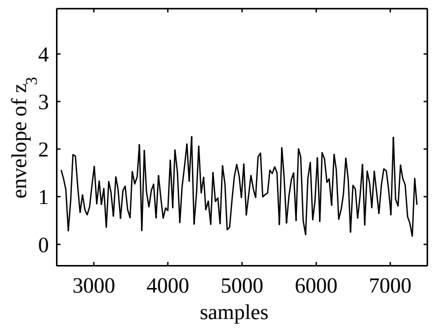

Figure 5: Envelope of a noisy separated output

and 10 accelerometers are placed on the structure. They record mixtures of all the sources. The figure 8 represents the spectrum of one observation in the band [0,1000]Hz. The method previously detailed in the paper has been applied on these data. The interspectral matrices are estimated on 200 blocks of 128 samples with 64 sample overlapping. Two sources are estimated. We show in figure 9 the spectrum of one of the two estimated sources. It is principally constituted of harmonic frequencies of fondamental 50Hz, which represents the contribution of the rotating system from only one test bed. This result proves the efficiency of the algorithm (separation step and Wiener filtering) on experimental noisy mixtures.

8. CONCLUSIONS

We consider here the issue of harmonic source separation in convolutive and noisy mixtures. With this method, the noises may be spatially correlated and their distributions need not be known. We propose a separation process relying uniquely on second order statistics. By exploiting interspectral matrices of time delayed observations, we introduce a new estimator of the decorrelation matrix and the signal subspace. After whitening, the delayed observations are used again to determine the Givens plane rotations that complete

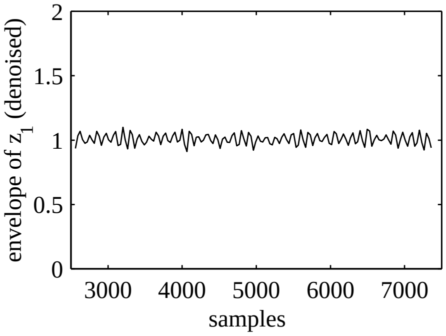

Figure 6: Envelope of a denoised output

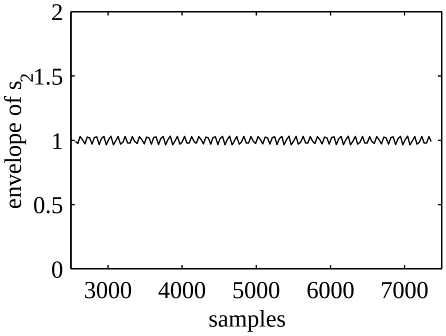

Figure 7: Envelope of the normalised source

the separation process. A noise reduction stage may thereby be pursued to improve the output Signal to Noise Ratio. The performance of all the steps is illustrated and quantified with a low Signal to Noise Ratio simulation and then with rotating machine vibration data.

9. REFERENCES

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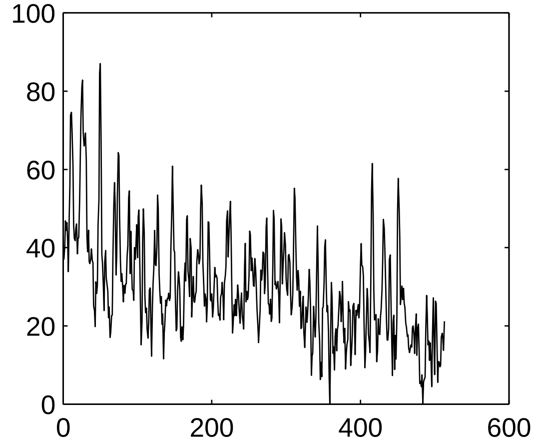

Figure 8: Spectrum of an observation in dB

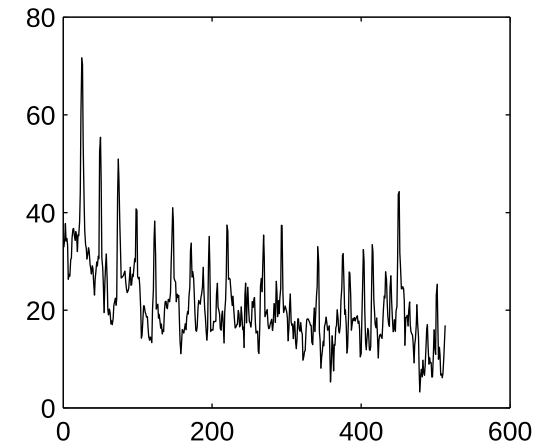

Figure 9: Spectrum of an estimated source in dB

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