MULTICHANNEL BLIND SOURCE SEPARATION AND BLIND EQUALIZATION USING FRACTIONAL SAMPLING

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ABSTRACT

Blind source separation (BSS) in communications applications is usually considered under the assumption that the observed or received signals are instantaneous linear mixtures of the individual sources without intersymbol interference (ISI). Based on a more general structure for multi-input multi-output channels, we present a method using fractional sampling and BSS to recover transmitted symbols in the presence of ISI. This method is different from traditional ways for dealing with ISI. It can be used successfully even if there are more sources than receivers. Simulation results are given, illustrating the good performance of the method and good agreement between theory and experiments.

Keywords: Blind source separation, Intersymbol interference, Fractional sampling

1. INTRODUCTION

Blind Source Separation is the process of recovering a set of independent signals when only some unknown additive mixtures are observed. Many interesting applications of BSS have been discussed, and BSS and more generally independent component analysis (ICA) continues to be actively investigated [1], [2], [3]. Yang [4] has recently addressed the problem of recovering the input sequence of a SISO channel with a discrete-time finite alphabet input and a continuous-time output, where BSS using fractional sampling was applied to achieve blind equalization of an FIR channel. Some interesting and useful results are given in [4], which include requirements on the oversampling factor and the sample size along with some performance comparisons with other blind equalization techniques. It is also shown in [4] that some degree of correlation between adjacent symbols may be tolerated, even though the theory is based on independent symbol sequences.

In this paper, we propose a more general linear distorting channel with multiple inputs and outputs. In the more general scenario, independent discrete symbols from finite alphabets produce complex amplitude-modulated carrier pulses at multiple transmitters. These pulses pass through a mixing channel, characterized by a channel impulse response matrix. As a special case of this general channel model, the standard setting of a BSS problem represents the mixing channel as a constant matrix and the received samples are instantaneous linear additive mixtures of the original symbols. To obtain separation and equalization in presence of ISI, we incorporate fractional sampling into BSS to recover the original symbols. A useful feature of the technique we discuss in this paper is that it allows for more sources than receivers. Discussions on estimating the number of sources and selecting best recovered signals in case of redundancies are also given in the paper.

The normalized Equivariant Adaptive Source Separation (EASI) algorithm [5] is used in this paper.

2. SOURCE SEPARATION OF INSTANTANEOUS MIXTURES

2.1. BSS Model

The block diagram in Fig. 1 shows a general adaptive BSS scheme for the standard model of instantaneous additive sources.

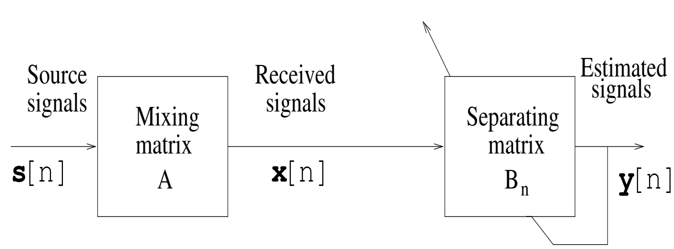

Figure 1: Adaptive BSS of instantaneous additive mixtures

The received discrete-time signal model is that of an m-dimensional time series $\mathbf{x}[n] = (x_1[n] \cdots x_i[n] \cdots x_m[n])^T$ of the form:

$$\mathbf{x}[n] \quad = \quad \mathbf{A}\mathbf{s}[n] \tag{1}$$

where

$$\mathbf{s}[n] \quad = \quad \left(\begin{array}{ccccc} s_1[n] & \cdots & s_i[n] & \cdots & s_k[n] \end{array} \right)^T$$

The channel characteristic between $\mathbf{s}[n]$ and $\mathbf{x}[n]$ is defined by the constant mixing matrix $\mathbf{A}$ of size $m \times k$; there are k sources and m receivers. Here we require $m \geq k$, which means the number of receivers should be no less than the number of sources.

The objective is to get a separating matrix such that $\mathbf{y}[n]$ consists of individually scaled and possibly permuted versions of $\mathbf{s}[n]$:

$$\begin{aligned} \mathbf{y}[n] &= \mathbf{B}_n\mathbf{x}[n] = \mathbf{B}_n\mathbf{A}\mathbf{s}[n] \\ &= \mathbf{C}_n\mathbf{s}[n] \end{aligned} \tag{2}$$

where

$$\mathbf{C}_n \stackrel{\text{def}}{=} \mathbf{B}_n\mathbf{A} \tag{3}$$

Ideally, for n large enough, $\mathbf{B}_n$ has converged to a matrix $\mathbf{B}$, and $\mathbf{C} = \mathbf{B}\mathbf{A}$ is very close to an identity matrix; more generally, $\mathbf{C}$ is a permutation matrix with arbitrary scaling for each output.

The standard assumptions made in BSS are:
Assumption 1: Matrix $\mathbf{A}$ is full rank with $m \geq k$;
Assumption 2: The individual source processes $\{s_i[n]\}$, $i = 1, \cdots, m$ are mutually statistically independent.

In order to recover the source signals under the presence of intersymbol interference, we also make the assumption:
Assumption 3: Source signals are i.i.d. sequences.

2.2. The EASI Algorithm

The normalized Equivariant Adaptive Source Separation (EASI) algorithm [5] for the $\mathbf{B}_n$ has the form:

$$\begin{aligned} \mathbf{B}_{n+1} &= \mathbf{B}_n - \lambda_n \left[\frac{\mathbf{y}[n]\mathbf{y}[n]^T - \mathbf{I}}{1 + \lambda_n \mathbf{y}[n]^T\mathbf{y}[n]} \right. \\ &\left. + \frac{\mathbf{g}(\mathbf{y}[n])\mathbf{y}[n]^T - \mathbf{y}[n]\mathbf{g}^T(\mathbf{y}[n])}{1 + \lambda_n |\mathbf{y}[n]^T\mathbf{g}(\mathbf{y}[n])|} \right] \mathbf{B}_n \end{aligned} \tag{4}$$

where λ_n is the adaptation step size, and $\mathbf{g}(\cdot)$ is a component-wise nonlinear odd function for which design guidelines are available.

3. FRACTIONAL SAMPLING AND SEPARATION/EQUALIZATION FOR LINEAR MIXING CHANNELS

3.1. The Case of Two Sources and Two Receivers

For simplicity, a two-source model is used in the following. Here $f_{11}(t)$, $f_{12}(t)$, $f_{21}(t)$ and $f_{22}(t)$ are the equivalent channel mixing impulse responses. The received two-dimensional signal is obtained as a convolution of the transmitted symbols and these mixing impulse responses:

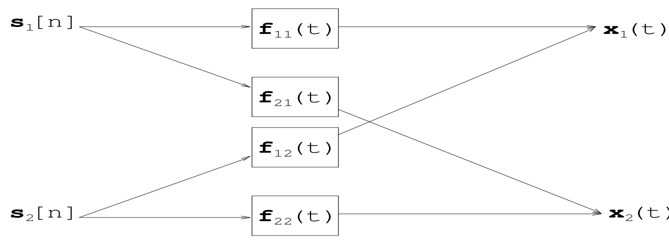

Figure 2: A general model of two-input two-output mixing channels

$$\mathbf{x}(t) \quad = \quad \sum_{n=-\infty}^{\infty} \mathbf{F}(t - nT) \cdot \mathbf{s}[n] \tag{5}$$

where

$$\begin{aligned} \mathbf{F}(t) &= \begin{pmatrix} f_{11}(t) & f_{12}(t) \\ f_{21}(t) & f_{22}(t) \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{f}_1(t) \\ \mathbf{f}_2(t) \end{pmatrix} \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{x}(t) &= \left(\begin{array}{cc} x_1(t) & x_2(t) \end{array} \right)^T \\ \mathbf{s}[n] &= \left(\begin{array}{cc} s_1[n] & s_2[n] \end{array} \right)^T \end{aligned}$$

and T is the symbol period. Here $\mathbf{f}_1(t)$ and $\mathbf{f}_2(t)$ are the row vectors of matrix $\mathbf{F}(t)$.

If symbol-rate sampling is applied at the receivers, then the sample vector taken at $t = lT$ is

$$\begin{aligned} \mathbf{x}[l] &= \mathbf{x}(lT) \\ &= \sum_{n=-\infty}^{\infty} \mathbf{F}((l-n)T) \cdot \mathbf{s}[n] \end{aligned} \tag{7}$$

In the presence of ISI, the received signals are linear mixtures not only of current independent symbols from different sources but also of the adjacent symbols from the same sources. If we treat this problem as a standard source separation problem with instantaneous mixtures only, then the *separated* results would be convolutive mixtures of adjacent symbols of each source separately. Using fractional sampling combined with source separation, it is possible in the general case to recover the original sources directly as long as we have enough information via oversampling.

Let us make the following assumption about the extent of ISI in the channels:

$$f_{ij}(t) = 0, \quad |t - D_j| \geq L_j T \ \ (L_j > 0),\ i,j \in \{1,2\} \quad (8)$$

where D_j is the centering point which gives the minimum integer L_j for each $f_{ij}(t)$. Then the *contributions* from source j with $j = 1,2$ can come from a maximum of $2L_j$ symbols at any receiver sampling point for different receivers. If we consider the set of samples over a T interval and allow different sampling offsets within a T interval at different receivers, we get *contributions* from a maximum of $2L_j + 2$ symbols from source j.

Let us assume different sampling offsets at different receivers. To be more conservative, let us assume $L = max(L_1, L_2)$. Fractionally sampling at rate M/T with $M > 2L+1$, the i-th received signal $x_i(t)$ at $t = lT + \frac{jT}{M} + \tau_i$ with $j = 0, 1, \cdots, 2L+1$, gives

$$x_i(lT + \frac{jT}{M} + \tau_i) = \sum_{n=l-L}^{l+L+1} \mathbf{f}_i((l-n)T + \frac{jT}{M} + \tau_i)\mathbf{s}[n] \quad (9)$$

$l = 0, 1, \cdots$, where τ_i is some unknown sampling offset at the i-th receiver, and $M > 2L+1$. We assume that $\tau_i \in [0, T)$ since any sampling offset outside this range can be accommodated by a shift in the time origin by a multiple of T.

Let

$$\mathbf{F}_j[l] \stackrel{\text{def}}{=} \left(\ \mathbf{f}_1(lT + \tfrac{jT}{M} + \tau_1)^T \ \mathbf{f}_2(lT + \tfrac{jT}{M} + \tau_2)^T \ \right)^T \quad (10)$$

$$\mathbf{r}_j[l] \ \stackrel{\text{def}}{=} \ \left(\ x_1(lT + \tfrac{jT}{M} + \tau_1) \ \ x_2(lT + \tfrac{jT}{M} + \tau_2) \ \right)^T \quad (11)$$

then

$$\mathbf{r}_j[l] \ = \ \sum_{n=-L-1}^{L} \mathbf{F}_j[n]\mathbf{s}[l-n] \qquad (12)$$

Redundant information about the source signals is contained in $\mathbf{r}_j[n]$ because of the oversampling, as can be seen from:

$$\begin{pmatrix} \mathbf{r}_0[l] \\ \vdots \\ \mathbf{r}_{2L+1}[l] \end{pmatrix} = \mathbf{G} \cdot \begin{pmatrix} \mathbf{s}[l-L] \\ \vdots \\ \mathbf{s}[l+L+1] \end{pmatrix} \quad (13)$$

with

$$\mathbf{G} \ = \ \begin{pmatrix} \mathbf{F}_0[L] & \cdots & \mathbf{F}_0[-L-1] \\ \vdots & & \vdots \\ \mathbf{F}_{2L+1}[L] & \cdots & \mathbf{F}_{2L+1}[-L-1] \end{pmatrix} (14)$$

of dimension $2(2L+2) \times 2(2L+2)$.

The received signals can be oversampled uniformly as above or non-uniformly but consistently in each T interval as long as there are enough samples within each interval T. Since $\mathbf{G}$ is a constant matrix, this will be a typical source separation problem if all the $2(2L+2)$ components in $\mathbf{s}[l-n]$ with $n = -L-1, \cdots, L$ are independent and $\mathbf{G}$ is of full rank. By the statistical independence of *Assumptions* 2 and 3, we conclude that for each l all the elements in $\mathbf{s}[l-n]$ for $n = -L-1, \cdots, L$ are indeed independent and identically distributed. Under these conditions, for the case shown in Fig. 2, the results of applying a BSS algorithm such as normalized EASI should yield a maximum of $2L+2$ symbols from each of the sources s_1 and s_2. The recovered sources are possibly amplitude scaled versions of the original source signals and the recovered constellations may be rotated. In addition, the recovered signals from the two sources may not be aligned on the same symbol interval, that is, the symbols of the first and second sources cannot be put in their original time correspondence.

We can generalize Eq.(13) for k sources and m receivers with $k \leq m$. Thus $\mathbf{G}$ will be a $m(2L+2) \times k(2L+2)$ matrix. If there are more sources than receivers with $k > m$, then in order to recover the original sources, we need to obtain $\lceil \frac{k}{m} \rceil \times (2L+2)$ samples per symbol period from each receiver. Therefore, our method allows for more sources than recievers. The case addressed by Yang in [4] is the $k = 1$ and $m = 1$ case. We can further extend Yang's method in [4] to the case where there are multiple sources and multiple receivers. In order to recover the original sources, it is necessary for the generalized Yang's method that the oversampling factor be chosen so that $M > k/m$. Our generalization of Yang's method also allows for more sources than receivers.

4. IDENTIFYING THE SOURCES AFTER BSS

As the analysis and our example in the next section show, the original sources are recovered with possibly multiple redundancy. Therefore, we have two issues to deal with. One is to recognize the sets of separated outputs that represent each source, and the other is to find the best signal among each set.

If there are m receivers and if samples are taken at a rate of M/T at each receiver, then the total number of samples for all the receivers is $N = mM$ within every symbol period T. Based on the samples in the n-th symbol period, the recovered signals are $y_i[n], i = 1, \cdots N$. Let

$$\mathbf{u}_i \ = \ (y_i[n] \ \cdots \ y_i[n-N+1]) \qquad (15)$$

with $i = 1, \cdots N$, which represents the values of one of the N separated signals.

Consider the case where the vectors $\mathbf{u}_i$ in (15) correspond to the perfect recontructions of the original sources. If any two vectors $\mathbf{u}_k$ and $\mathbf{u}_j$ with $k \neq j$ are the recovered sequences of two different signal sources, then their cross-covariance matrix will be the zero matrix. On the other hand, if $\mathbf{u}_k$ and $\mathbf{u}_j$ with $k \neq j$ represent the recovered sequences of the same source, then their cross-covariance matrix may not be the zero matrix. This is because the vectors $\mathbf{u}_k$ and $\mathbf{u}_j$, although not aligned on the same set of signaling periods, may have elements in common. Eq.(15) shows row-vectors $\mathbf{u}_i$ with $i = 1, \cdots, N$ have dimension N, however, we can choose any other dimension as long as $\mathbf{u}_k$ and $\mathbf{u}_j$ with $k \neq j$, representing the same source, have at least one symbol in common. Therefore, based on this property, we can design a method to group all the recovered signals into different sets. Each set contains signals representing the same source. The actual number of sources can be estimated based on the number of sets we obtain after the grouping. Singular value decomposition can be applied to group the recovered signals.

After all the recovered signals have been grouped into different sets, one possible method for picking a good recovery of the original source utilizes higher order statistics. We can pick the signal with the maximum sample kurtosis within each set. This implies that we pick the signal which has the most compact clusters within each set.

5. SIMULATIONS

We present here representative simulation results for the method discussed in this paper. Example 1 is given for the two-input, two-output general channel in Fig. 2. Example 2 is given for a two-input, one-output channel. In both cases the two sources are a 4QAM and a 16QAM source. Each source generates 5000 symbols in Example 1, and 10000 symbols in Example 2. We show the last 1500 recovered symbols in each of our simulation results. White Gaussian noise is added in each example with $SNR = 25dB$. The SNR is computed as the ratio of the average power of the source constellations to the noise power.

5.1. Example 1

The two sequences of source symbols are transmitted by squared-half-cosine carrier pulses defined as follows:

$$p_1(t) = p_2(t) = \begin{cases} \cos^2(\pi t/(2T)) & \text{if } -T < t < T \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

These pulses pass through a mixing channel. This mixing channel is represented as a constant matrix. The symbols from each source are transmitted with timing offsets ($\Delta_1 = 0.32T$ and $\Delta_2 = 0.44T$) relative to the time origin. The continuous time mixing impulse response functions defined in (6) are

$$\begin{aligned} f_{11}(t) &= c_{11}p_1(t-\Delta_1) && (17)\\ f_{21}(t) &= c_{21}p_1(t-\Delta_1) && (18)\\ f_{12}(t) &= c_{12}p_2(t-\Delta_2) && (19)\\ f_{22}(t) &= c_{22}p_2(t-\Delta_2) && (20) \end{aligned}$$

Here T is the symbol period and c_{11}, c_{21}, c_{12} and c_{22} are randomly generated complex numbers. In our simulation for this example, we had $c_{11} = 0.8558 + 0.4393i$, $c_{21} = -0.7784 - 0.0321i$, $c_{12} = -0.1636 - 0.5533i$ and $c_{22} = -1.4002 + 0.8690i$. Since $L_1 = 1, L_2 = 1$ in this situation, $L = max(L_1, L_2) = 1$

Using our method, we choose the oversampling factor $M = 4$. Samples are taken at $t = lT + \frac{j}{M}T + \tau_i$ with $j = 0, \cdots, 2L+1$, $l = 0, 1, \cdots$ and $i = 1, 2$. The sampling offsets at each receiver are $\tau_1 = 0$ and $\tau_2 = 0.98T$ respectively. Four samples are taken at each receiver every symbol period. The recovered signals are shown in Fig. 3.

Using the generalized Yang's method, we choose the oversampling factor $M = 2$. Samples are taken at $t = lT + \frac{j}{M}T + \tau_i$ with $j = 0, 1$, $l = 0, 1, \cdots$ and $i = 1, 2$. The sampling offsets at each receiver are $\tau_1 = 0$ and $\tau_2 = 0.98T$ respectively. Two samples are taken every symbol period. All the samples taken over three consecutive symbol periods are used for one iteration in the BSS algorithm. As mentioned earlier, the generalized Yang's method needs to recover four more symbols than ours. Due to the limit on the length of the paper, simluations results for Yang's method are not included here.

Both methods are combined with the same EASI algorithm in (4) with $\lambda_n = 0.003$ and $\mathbf{g}(\cdot)$ as a componentwise cubic function, and are run for the same number of iterations. Our simulation shows that the results for both methods become noisier if we increase the value of λ_n. On the other hand if we decrease the value of λ_n, we will get a slower convergence rate. We found the generalized Yang's method was more sensitive to the choice of λ_n compared to our method. The recovered signals using our method form the constellations shown in Fig. 3, and are less spread out compared to those for the generalized Yang's method. We note that our method needs to implement a higher sampling rate.

5.2. Example 2

Now there is only one receiver. One of the carrier pulses $p_1(t)$ is squared-half-cosine defined in (16). The other is a double-exponential-decaying pulse defined as follows:

$$p_2(t) = \begin{cases} \frac{te^{(1-t/T)}}{T} & \text{if } -T < t < T \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

The two channel impulse responses are defined in the following.

$$h_{11}(t) = \begin{cases} (e^{-2t/T} - e^{-t/T})/T & \text{if } 0 < t < 2T \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$h_{12}(t) = \begin{cases} 7e^{-t/T} \cdot \cos(3t/T) & \\ \quad -5e^{-t/T} \cdot \sin(3t/T)/12 & \\ \quad +23e^{-3t/T} \cdot \cos(t/T)/4 & \\ \quad +25e^{-3t/T} \cdot \sin(t/T)/4 & \text{if } 0 < t < 2T \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

The equivalent channel mixing impulse response functions are defined by

$$\begin{aligned} \mathbf{F}(t) &= \begin{pmatrix} f_{11}(t) & f_{12}(t) \end{pmatrix} \\ &= \begin{pmatrix} p_1(t) \star h_{21}(t) & p_2(t) \star h_{12}(t) \end{pmatrix} \end{aligned} \tag{24}$$

We have $L_1 = L_2 = 2$ in this case.
Therefore, $L = max(L_1, L_2) = 2$. Samples are taken at $t = lT + jT/M$ with $M = 5$, $j = 0, \cdots, 2L$ and $l = 0, 1, \cdots$. Fig. 4 shows the recovered the symbols from two sources with sampling rate at $5/T$.

6. CONCLUSION

In this paper, fractional sampling and BSS are applied to recover input source symbol streams for a general structure of mixing channels with multiple inputs and multiple outputs. We have shown that fractional sampling can solve the problem of recovering independent source signals from their mixtures in presence of ISI. Moreover, fractional sampling makes it possible and to recover the original sources when there are more source than receivers. In essence, the sensitivities to both ISI and the number of receivers, which are among the difficult problems in BSS, have been avoided by applying fractional sampling.

As shown in both analysis and simulations, fractional sampling combined with source separation algorithms are very useful under real situations and can be applied to both blind source separation and blind deconvolution.

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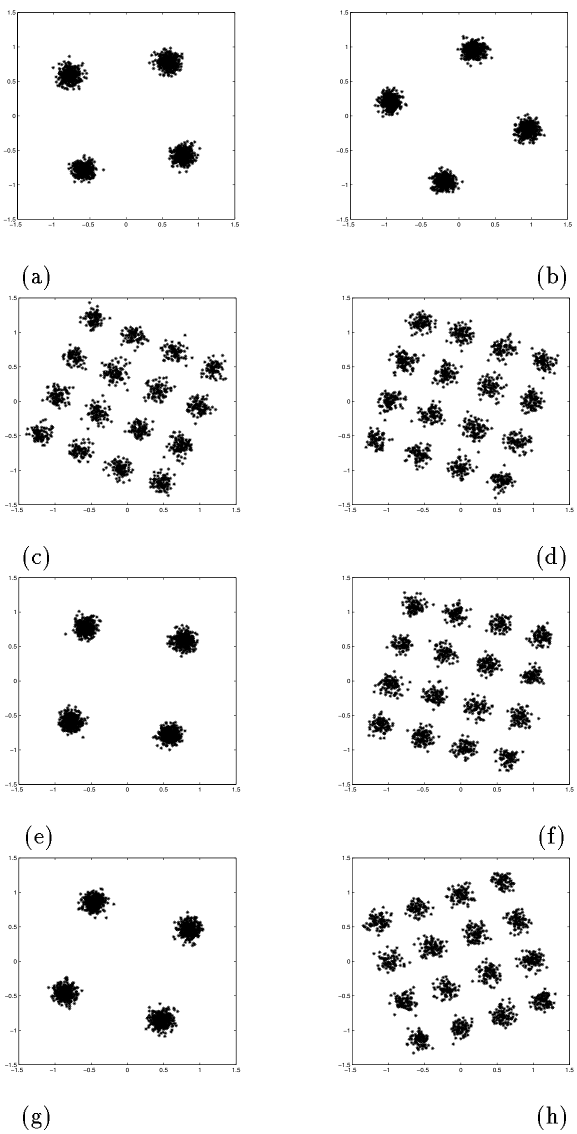

Figure 3: **Example 1:** Separating the mixtures of one 4QAM and one 16QAM signals; Squared-half-cosine carrier pulses are used for transmission. SNR=25dB. From (a) to (h) are the results of separation using our method.

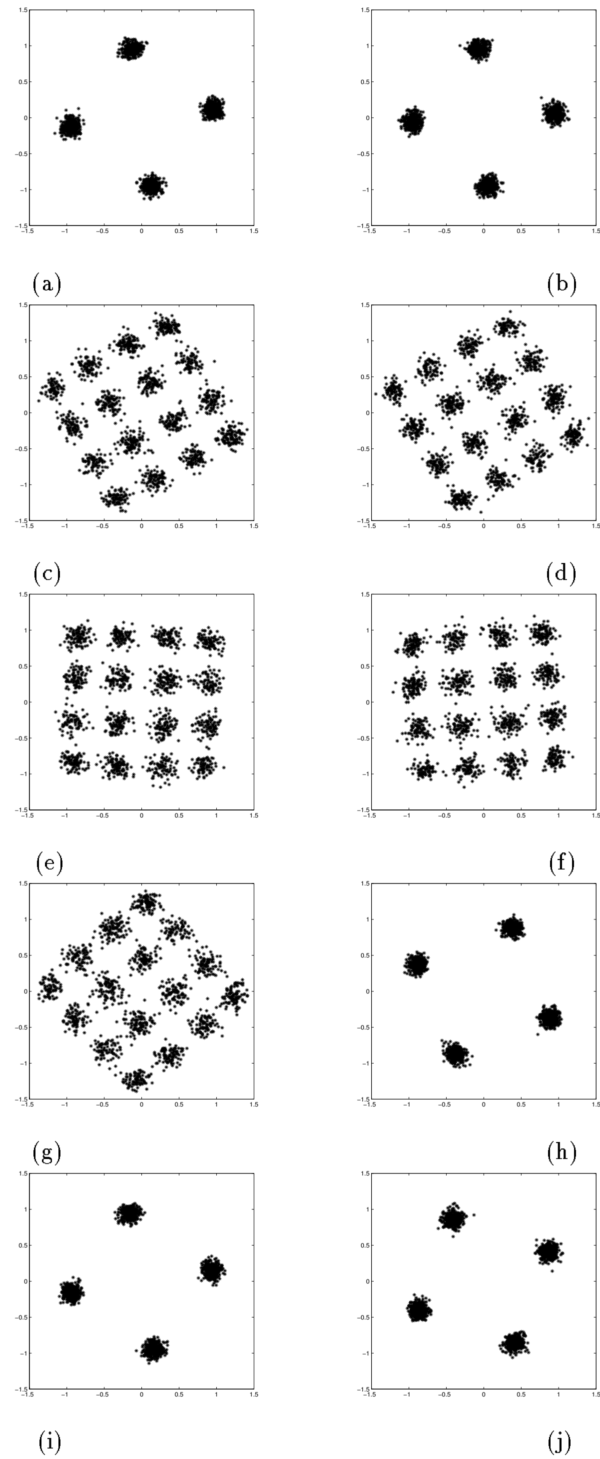

Figure 4: **Example 2:** Separating the convolved mixtures of one 4QAM and one 16QAM signals; Only one receiver is used. Squared-half-cosine and double-exponential-decaying carrier pulses are used for transmission. SNR=25dB. From (a) to (j) are the results of separation using our method.