ENHANCING USEFULNESS OF APERIODIC STOCHASTIC RESONANCE IN THE IMPROVEMENT OF BSS APPLIED ON AN EDDY CURRENT SENSOR NONLINEAR RESPONSE

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ABSTRACT

This paper reports a form of aperiodic stochastic resonance (ASR) observed in a specific experimental context. This term is attributed to a phenomenon wherein the response of a nonlinear system to a periodic or aperiodic (deterministic or random) input is optimized by the assistance of noise. Here we provide some elements of how ASR can be incorporated in signal processing methods. By coupling it with BSS, we improve the frequency response of an eddy current proximity sensor. Experimental results are presented to validate our approach.

1. INTRODUCTION

An eddy-current sensor is a contactless device which consists in a coil of wire excited by a high frequency current. The working principle can be resumed as follows: when the coil is excited by a sinusoidal current, a magnetic field is generated around it. The introduction of a nonmagnetic conducting object (target) into this field induces eddy currents on its surface, creating then another magnetic field which tends to oppose the coil's one (Lenz's law). The mutual inductance between the coil and the target varies then with the gap probe-target, and this variation is exploited by a signal conditioner in order to provide a measurement of this distance. This can be done in different manners, and particularly, by introducing the coil in an oscillator, which central frequency changes with the mutual.

A real problem with the use of eddy current sensors is that they are sensible to temperature variations. The aim of our research is to attenuate the effects that can have temperature changes on a distance measurement provided by such a sensor made in our laboratory, via source separation methods. Because previous results with linear instantaneous BSS were not precise enough due to noise and inherent nonlinearity of the response [1], we present here results obtained by using some kind of ASR to improve the separation quality. This work follows the one presented in [2], in which we used the a priori knowledge of temperature in a similar manner.

The paper is organized as follows. Section 2 presents the historical background of Stochastic Resonance (SR), some definitions related to ASR and reminds the measures used to characterize it. After a brief introduction to BSS, section 3 explain how it can be applied to our sensor response. Section 4 shows under which form ASR appears in this particular frame and establishes the relationship between ASR and BSS, while numerical results are presented in order to quantify ASR using information measure and to show its interest when it is coupled with a BSS method. A discussion in Section 5 concludes this work.

2. STOCHASTIC RESONANCE: SOME HISTORY

What is Stochastic Resonance? Does exist a unique definition of this phenomenon? So far, researchers working on it, agree that SR is a nonlinear phenomenon (effect) of noise-enhanced signal transmission. It has been observed for the first time, about twenty years ago now. The notion of SR was originally proposed as a possible explanation for periodic recurrences in global climate dynamics ([3]). Since, this paradoxical effect has been extended and observed in a wide variety of nonlinear systems, such as electronic circuits ([4], [5]) or sensory neuron ([6], [7]).

SR arises under various forms, according to the signals and the nonlinear transmission systems, and according to the information measure which is improved

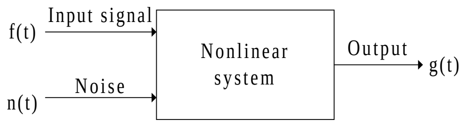

Figure 1: A general scheme for RS : the coherence between the input stimulus and the system response is optimized with the add of noise

via addition of noise [8]. How can one define and quantify a SR effect observed experimentally?

2.1. Aperiodic Stochastic Resonance (ARS): definitions

Figure 1, is a general scheme for RS, in which the flow of information through a system is optimized by the presence of a particular level of noise. The input signal is generally periodic and the noise is often white and Gaussian, but recently, RS deals also with aperiodic (arbitrary) inputs [9]. The RS effect is composed by four basic ingredients: (i) an input noise $n(t)$ of various distributions (Gaussian, laplacian, uniform ...), (ii) an input stimulus $f(t)$, deterministic periodic or aperiodic, that carries out the information we seek to extract from noise, (iii) a generally nonlinear transmission system which produces the output signal $g(t)$ after being excited by $s(t)$ and $n(t)$ and (iv) a performance or information measure able to quantify this phenomenon, in terms of likelihood between the output and the input signal.

Definition. *We will refer to a phenomenon as Stochastic Resonance, each time it is possible to increase the performance measure by increasing the level of input noise [10].*

Note also, that if SR was limited to the treatment of systems with periodic input signals, its applicability to real world signals, often not periodic, could be compromised. So, we coin the term Aperiodic Stochastic Resonance (ASR) for characterizing SR-type behavior in excitable systems with aperiodic inputs [11], which can be *deterministic* or *random* signals [12].

2.2. Performance measures quantifying ASR

A classical periodic SR signature is a signal-to-noise ratio (SNR) that is not monotone. SNR computed from the power spectrum, but also Shannon mutual information, are periodic SR measures defined in the frequency domain. Both of these methods assess the coherence of the system response with the input signal frequency and thus, they are clearly inappropriate for systems with aperiodic inputs.

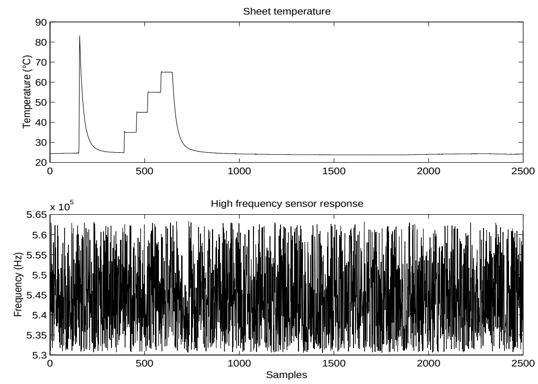

Figure 2: Sheet temperature and sensor response

Cross-correlation measures: these "shape matchers" measure SR performance when inputs are not periodic signals. They define cross-correlation measures for the input signal $f(t)$ and the system response as follows:

$$\begin{aligned} C_0 &= \max\left\{\overline{f(t+\Delta)g(t)}\right\}, \qquad (1) \\ C_1 &= \frac{C_0}{\left[\overline{f^2(t)}\right]^{1/2}\left[\overline{(g(t)-\overline{g(t)})^2}\right]^{1/2}}, \end{aligned}$$

where $f(t)$ is the aperiodic (zero mean) input signal, Δ is a time lag and the overbar denotes an average over time. From a signal processing perspective, maximizing the *normalized power norm* C_1 corresponds to maximizing the shape matching between the input stimulus $f(t)$ and the system response $g(t)$, whereas maximizing the *power norm* C_0 corresponds to taking account of both signal amplification and shape matching ([11], [13]).

Other ASR performance measures proposed in [10] for bidimensional signals (images) are:

$$R = \frac{\overline{fg}}{\sqrt{\overline{f^2 g^2}}} \qquad \text{and} \qquad C = \frac{\overline{fg} - \overline{f}\,\overline{g}}{std(f)std(g)}. \qquad (2)$$

R and C define the *normalized cross-correlation* and the *normalized cross-covariance*, respectively. These measures have been used to identify an RS effect in spatial signals in optics. We will show further, how a performance measure depends on the context or on the aim of the experimental frame, with respect to the definition given in the previous subsection.

3. BSS BACKGROUND

The objective of source separation is to separate mixed sources $s(t)$ issued from real phenomena, from a given

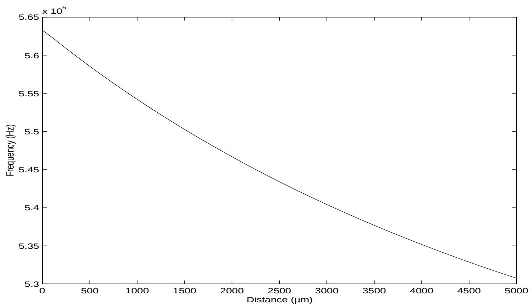

Figure 3: Classical form of a lift-off curve

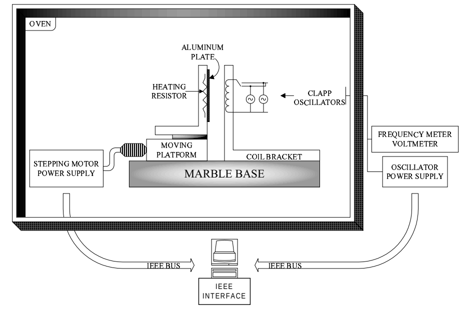

Figure 4: Experimental setup for distance/temperature separation

number of sensors responses which receive an instantaneous mixing $x(t)$ of these sources. This is the simplest form of the source separation problem. When the separation doesn't assume any specific a priori knowledge on the structure of the sources and relies only on their statistical independence, this problem is called Blind Source Separation (BSS) and was introduced by Hérault et Jutten [14]. BSS is often related to ICA, that can be considered as the underlying mathematical technique for solving a BSS problem and was first developed in a rigorous manner by Pierre Comon [15] as a generalization of the technique of Principal Component Analysis (PCA), concept that provides only decorrelated components. We will consider the noisy instantaneous model, that can be described in mathematical terms as follows:

$$x(t) = As(t) + \nu(t), \tag{3}$$

where x is an $m \times 1$ vector of observations, s is an $n \times 1$ vector of sources with statistically independent components and A is the unknown constant mixing matrix, i.e. we suppose that s is stationary over time t. Here, ν is an additive $m \times 1$ noise vector independent with s, often considered to be Gaussian. It is well known now that if the noise is really Gaussian, the sources can be recovered at best up to a scale, sign and permutation. If this is not the case, performances can decrease, but one can apply classical BSS methods if no a priori knowledge is available about the noise statistics.

3.1. Eddy current sensor response and BSS

In our problem, the first source to estimate will be naturally the distance d. Then, considering that under normal conditions the sensor is used at a reference temperature $T = \theta_{ref}$, we choose the temperature gap $\Delta T = \theta - \theta_{ref}$ as the second source. We can not consider directly the temperature as a source because of the thermal dilations that cause the plate and coil width to increase, creating then a relation between the measured distance and the plate temperature and leading thus to a dependency. We have used a *multi-frequency sensor* made of two or three Clapp-oscillators functioning over different ranges of frequency with the same coil element. The underlying idea being that for different frequencies, the penetration depth of the eddy currents in the target is not the same and, in the same manner, the temperature effect on sensor responses is different. Thus, each oscillator provides a different mixing of our two sources.

3.2. Mixing linearization

In order to calibrate the sensor, we make what we call a *reference lift-off curve*, providing us the frequencies of each oscillator corresponding to the distances measured by an external interferometric laser system (Figure 3). For one oscillator, this response $f^{\theta_{ref}}$ is typically non-linear and decreases exponentially with increasing coil-target distance d. Empirically, it can be written as

$$f^{\theta_{ref}} = f_0^{\theta_{ref}} (1 - \beta^2 \exp(-\alpha_1 d - \alpha_2 d^2)^2)^{-1/2}, \tag{4}$$

$f_0^{\theta_{ref}}$ being the oscillator frequency without target. For a given distance d, we have also observed that the frequency response f^{θ} of an oscillator influenced by changes in target temperature could be approximated by the empirical model:

$$f^{\theta} = f^{\theta_{ref}}(d) + \Delta T[a_1 + a_2 \exp(-(1 + a_3 \Delta T)d]. \tag{5}$$

This mixing of our two sources created in each oscillator is highly nonlinear and can not be used as it is if we want to follow the model (3). We also have to linearize it at best by reversing eq. (4) or better by fitting the curve with a feed forward neural network, in order to get an estimated distance for a given frequency. By making the assumption that this method brings approximately a linear instantaneous mixing embedded in noise, inherent to any real world problem and in

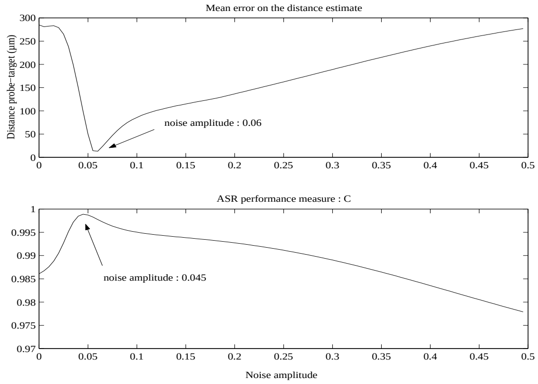

Figure 5: Evolution of C and distance error

our case additionally due to modelization errors, we have tried to apply a BSS algorithm on a random distance signal influenced by temperature variations of the plate. The first results were promising because we could restitute a distance and a temperature profile, but these ones were not precise enough because of subsisting non-linearities [1]. However, the assumption of the linear mixing is not totally false and we can show that it can be a real linear one if we constrain the sensor response to follow their mean behavior [16]. When using a simple linear model, a solution that give good and accurate results is to use the a priori knowledge of the temperature [2], but this is restrictive because it implies the use of a temperature sensor. We thus developed another solution using noise-enhanced signal propagation, exhibiting then a stochastic resonance effect.

4. ASR COUPLED WITH BSS

From equation 4, our sensor frequency response can be transformed on a distance measurement, after estimation of the optimal parameters set $\widehat{w} = \{\widehat{\beta}, \widehat{\alpha}_1, \widehat{\alpha}_2\}$ using a reference lift-off curve. Consider now the system of Figure 6, where the input signal f is the frequency response of the eddy current sensor, n is an additive Gaussian white noise and the system output g is the distance coil-target estimated vector, after application of a BSS method. What is then inside the NL black box ? Obviously, this box contains a static nonlinearity,

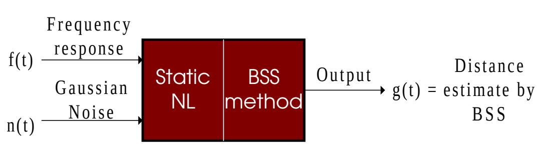

Figure 6: Experimental ARS scheme

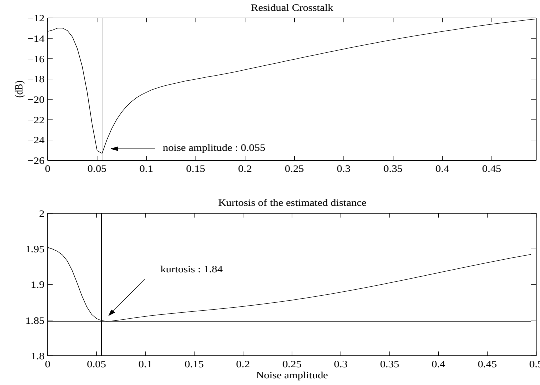

Figure 7: Evolution of RCT and kurtosis

that is the nonlinear model given by eq. (4) and a processing block, that is a BSS algorithm, such as JADE [17]. The main idea was to, somehow, noise-enhance the transmission of the input signal f, which contains the information about the distance d that we want to estimate, by applying a source separation method. The results provided by this method are quite surprising.

4.1. Experimental results

The experimental setup consists in a moving aluminum plate (target), warmed up on its back side, facing a flat spiral coil (the principal element of our sensor) which is engraved on a printed circuit board [18]. Two external devices (interferometer laser system and temperature sensor) provide measurements of d and θ. The whole device is also placed in an oven, regulated in temperature and humidity, so as to minimize the contribution of the other environmental changes, as shown in Figure 4. For our experiment, we used three oscillators with central $\{high, medium, low\}$ frequency values of $\{989, 499, 98\} KHz$, respectively. The target was moving in a $[0, 5000]\mu m$ range while its temperature was varying. The choice of distance in time is arbitrary within it and hence, its distribution is almost uniform. Figure 2 shows the sensor response at high frequency and the plate temperature profile.

After calibration of our sensor (§3.2), we applied JADE algorithm to a mixture vector with two components, which are distances deduced from the frequency response via the model of equation (4). But, before applying this nonlinear transformation, we added a gaussian white noise $n(t)$ with amplitude varying in $[0, 0.5]$ in only one frequency response component. Figure 8 summarizes this processing step: handwritten $\{f_1, f_2\}$ are two of the three available frequency responses of the sensor, their distance transforms $\{d_1, d_2\}$ being the mixture to the BSS, while the source separation algo-

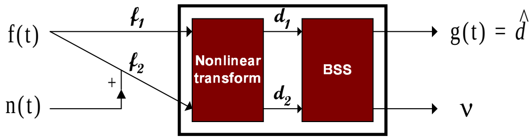

Figure 8: Processing flow

rithm provides two source estimates: $\widehat{d}$ which corresponds to the distance profile and ν, a noise vector which tends to white noise with increasing $n(t)$ noise amplitude, because the information about the temperature source θ is overwhelmed by the additive noise.

4.2. Is this ASR?

Various simulations have been realized taking all possible combinations for $\{f_1, f_2\}$. First, $n(t)$ was a Gaussian white noise generated with the same seed for each trial. For a given intensity level of the input noise, we have computed the performance measure C (§2.2) as well as the *Residual Crosstalk* (RCT) between the exact distance measurement and its estimate after BSS. RCT is a common performance criterion in source separation defined by (in dB):

$$RCT(s) = 10 \log_{10} \frac{E\left[(\widehat{s} - s)^2\right]}{E\left[s^2\right]}, \qquad (6)$$

where $\widehat{s}$ is the estimate of one source s. The more the similarity between s and $\widehat{s}$ increases the more RCT tends towards $-\infty$. As the source of interest in this particular context is d, we also computed at each trial, the estimation error by considering the mean of the absolute values of the difference between exact distance values and estimated ones provided by JADE, after centering and rescaling the amplitude levels of the source and its estimate.

Work presented in [2] dealt with the same input signals with a zero noise amplitude and the distance estimation error provided by JADE was around $300\mu m$ with regard to the choice of $\{f_1, f_2\}$. By adding noise as described above, this error dramatically decreases to $11,9\mu m$ for an 'optimal' noise amplitude of 0.06. This is also better than the error provided by the least perturbed sensor frequency response, that is $22\mu m$ and of the same order than the one provided after BSS using the a priori plate temperature knowledge.

The ASR performance measure C takes its maximum value for an optimum input noise amplitude of 0.045, in the case that f is the vector constituted by the medium and the high sensor frequency response (Figure 5). As to RCT, it takes its minimum value for an $n(t)$ of optimum amplitude value 0.055 (Figure 7), which is closer to the one corresponding to the minimum distance estimation error. The evolution curves of C and RCT match the characteristic curves of SR. Keeping in mind that there is no explicit expression for the power norm C_0 and that C is a possible but not unique ASR performance measure, we can use RCT as an ASR information measure well adapted to our experimental context and very well coupled with BSS methods. *With respect to the definition given in §2.1, the observed phenomenon characterizes one of the various forms of ASR.*

One interesting feature is the evolution of the kurtosis which is zero for a gaussian variable. Looking at the evolution of the separated distance kurtosis, one can see that it yields the value of the real distance signal (1.84) for the same input noise amplitude level than the optimum one in the RCT sense. Table 4.2 gives some results of our simulations.

	Noise amplitude	Error (μm)
$BSS(f_2, f_3)$	0	284.7
$C(f_2, f_3)$	0.045	99.2
$RCT(f_2, f_3)$	0.055	14.2
$BSS(f_1, f_3)$	0	290.6
$C(f_1, f_3)$	0.06	101.7
$RCT(f_1, f_3)$	0.07	23.02

Table 4.2: Numerical results

In this table, $\{f_1, f_2, f_3\}$ designates the $\{high, medium, low\}$ sensor frequency response, the column 'noise amplitude' gives the optimum input noise amplitude for the performance measures we used (which is 0 for classical BSS method) and the third column corresponds to the associated distance estimation error. From these results, it is obvious that the conditioning of the mixing is improving by using a noise-enhanced signal transmission method. Similar results were obtained when the Gaussian white noise was generated with different seeds for each trial. The evolution of both performance measures are more wavy, but by linear interpolation, one obtains the same SR characteristic curves evolution as above.

5. DISCUSSION

In this work we have demonstrated experimentally an ASR effect which, when coupled with a BSS method improves the response of an eddy current proximity sensor. Instead of using a classical BSS approach that provides not very accurate results due to the high nonlinearity of the mixing, Gaussian white noise is added to one of the components of that later, and we have seen that, for an optimal value of the input noise amplitude, the error on the estimation of the distance probe-target reaches a minimum. We have also shown, that the Residual Crosstalk can be a suitable ASR information measure, when the aim is to maximize it or minimize it according to the experimental context.

The results presented here prove the interest of this approach, as it dramatically improves the accuracy of the BSS distance estimate. Comparatively to previous non blind approaches or even filtering [2], there is no need to have an a priori knowledge about the sources and yet, the results obtained so, are of the same order of precision. This modern approach of ARS in terms of information measure could be applied in various ways in other domains of nonlinear signal processing which deal with real world signals.

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