DELAY ESTIMATION IN CDMA COMMUNICATIONS USING A FAST ICA ALGORITHM

Răzvan Cristescu[1], *Tapani Ristaniemi*[2], *Jyrki Joutsensalo*[3], *and Juha Karhunen*[1]

[1]Neural Networks Research Centre, Helsinki University of Technology
P.O.Box 5400, FIN-02015 HUT, Espoo, Finland
[2]Dept. of Mathematical Information Technology, University of Jyväskylä
P.O. Box 35, FIN-40351 Jyväskylä, Finland
[3]Signal Processing Laboratory, Tampere University of Technology
P.O.Box 553, FIN-33101 Tampere, Finland
`razvan@mail.cis.hut.fi`, `riesta@mit.jyu.fi`, `jyrkij@cs.tut.fi`, `Juha.Karhunen@hut.fi`

ABSTRACT

In this paper, we propose a method for timing acquisition in a code division multiple access (CDMA) communications system. The assumptions made are that the data signals corresponding to different users are statistically independent, and that a minor pilot sequence corresponding to the desired user is available. An independent component analysis (ICA) approach based on a fast fixed-point algorithm is used for simultaneous demodulation of the symbol sequence and finding of synchronization information of the user of interest. No additional pilot signal is used for the timing acquisition. Our semi-blind method performs better than conventional matched filter detector and CMOE receiver.

1. INTRODUCTION

Code Division Multiple Access (CDMA) is a data transmission technique for the third generation of mobile phones, based on spread spectrum methods. The CDMA system enables users to transmit over the same frequency band simultaneously. The main advantage of CDMA techniques over the more traditional TDMA and FDMA techniques based on time or frequency division is that they utilize better the overall transmission capacity, allowing more users in the system. The identification of a particular user in a CDMA system is done by his unique code.

In mobile communications systems, the required signal processing differs in the base station (uplink) from that in the mobile phone (downlink). In the base station, all the signals sent by different users must be detected, but there is also much more signal processing capacity available. The codes of the all the users are known but their delays are unknown. For delay estimation, one can use for example the simple matched filter [12, 14], subspace approaches [1, 13], or the optimal but computationally highly demanding maximum likelihood method [12, 14]. When the delays have been estimated, one can estimate the other parameters such as fading process and symbols [14].

In downlink (mobile phone) signal processing, each user knows only their own code, while the codes of the disturbing other users are unknown. There is less processing power than in the base station. Also the mathematical model of the signals differs slightly, since users share the same channel in the downlink communications. Especially the first two differences call for new, efficient and simple solutions.

In this paper, we propose a synchronization method based on independent component analysis (ICA) [9, 3, 11]. The ICA model is utilized mainly in blind separation of unknown source signals from their linear mixtures. In this application only the source signals, which correspond to the coefficients of the ICA expansion, are of interest. However, ICA can be used also for estimation of the basis vectors of ICA. In CDMA application, this means the delay estimation of the code sequences.

Experiments with simulated downlink CDMA data are presented in the paper. We apply the fixed-point algorithm [6], which is the fastest and simplest linear ICA algorithm to our knowledge [2]. A threshold element is then finally considered for determining the delay from the obtained ICA solution. Our experiments show that the proposed method has quite competitive synchronization cabability compared with the traditional matched filter [12, 14] and the recently reported Constrained Minimum Output Energy method [10, 8].

2. CDMA SIGNAL MODEL

The signal model studied in this paper is a downlink CDMA model with fixed multipaths. The data thus have the form

$$r(t) = \sum_{m=1}^{N} \sum_{k=1}^{K} b_{km} \sum_{l=1}^{L} a_l s_k(t - mT - d_l) + n(t) \quad (1)$$

Here a_l is the attenuation factor of the lth transmission path, called also path gain, containing the corresponding path power. The term b_{km} is kth user's mth symbol, and $s_k(\cdot)$ is kth user's chip sequence (spreading code), $s_k(t) \in \{-1, +1\}$, $t \in [0, T)$, $s_k(t) = 0$, $t \notin [0, T)$. The delay d_l corresponding to the lth path is assumed to be constant during the observation time, and $n(t)$ denotes the additive noise. The length of the chip sequence is C, and N is the number of bits in the observation interval.

The received data is first sampled. We collect C-length vectors from subsequent discretized equispaced data samples $r[n]$:

$$\mathbf{r}_m = [\ r[mC]\ r[mC+1] \ldots r[(m+1)C-1]\ ]^T \quad (2)$$

Then we can write [1]

$$\mathbf{r}_m = \sum_{k=1}^{K} [b_{k,m-1} \sum_{l=1}^{L} a_l \underline{\mathbf{g}}_{kl} + b_{km} \sum_{l=1}^{L} a_l \overline{\mathbf{g}}_{kl}] + \mathbf{n}_m \quad (3)$$

where $\mathbf{n}_m$ denotes the noise vector, and the 'early' and 'late' parts of the code vectors are respectively

$$\underline{\mathbf{g}}_{kl} = [\ s_k[C - d_l + 1] \ldots s_k[C]\ 0 \ldots 0\ ]^T \quad (4)$$

$$\overline{\mathbf{g}}_{kl} = [\ 0 \ldots 0\ s_k[1] \ldots s_k[C - d_l]\ ]^T \quad (5)$$

Here d_l is the discretized delay, $d_l \in \{0, \ldots, (C-1)/2\}$. The matrix $\mathbf{R} = [\mathbf{r}_1 \ldots \mathbf{r}_N]$ can be represented in the compact form

$$\mathbf{R} = \mathbf{GB} + \mathbf{N} \quad (6)$$

where the $C \times 2K$ dimensional mixing matrix $\mathbf{G}$ depends on the codes and path gains:

$$\mathbf{G} = [\sum_{l=1}^{L} a_l \underline{\mathbf{g}}_{1l}, \sum_{l=1}^{L} a_l \overline{\mathbf{g}}_{1l}, \ldots, \sum_{l=1}^{L} a_l \underline{\mathbf{g}}_{Kl}, \sum_{l=1}^{L} a_l \overline{\mathbf{g}}_{Kl}] \quad (7)$$

The $2K \times N$ matrix $\mathbf{B} = [\mathbf{b}_1 \ldots \mathbf{b}_N]$ depends only on the symbols transmitted by the users. Its column vectors are of the form

$$\mathbf{b}_m = [b_{1,m-1}, b_{1,m}, \ldots, b_{K,m-1}, b_{K,m}]^T \quad (8)$$

3. FAST ICA ALGORITHM

Independent component analysis (ICA) [9, 3, 11] is a recently developed technique whose goal is to represent a set of random variables as a linear transformation of statistically independent component variables. The unknown linear mixing, as well as the generating unknown independent components can be found by using ICA algorithms that utilize either higher-order statistics or time correlations of the observed variables. Under our assumption of independence of different users' transmitted symbol sequences, equation (6) represents the noisy standard linear ICA model [9, 3, 11] expressed in matrix form. In the model (6), $\mathbf{B}$ is a matrix which has as its columns the source vectors, $\mathbf{R}$ is the respective observation matrix whose columns consist of the observed data vectors, and $\mathbf{G}$ is the unknown mixing matrix.

A reliable and fast method for extracting independent components from the observed data is the fixed-point ICA algorithm, also called FastICA, which has been introduced in [6] and is further developed in [5]. It is a semi-neural method which uses higher-order statistics either directly or indirectly via nonlinearities to extract the independent components. Our method for timing acquisition is based on the ability of the ICA algorithm to estimate simultaneously both the symbol sequences and the mixing matrix of the ICA model. Such an approach was applied to CDMA data in our earlier paper [7] for the first time to our knowledge.

There exist several somewhat different versions of the fixed-point algorithm [6, 5]. We use it in the following form. First, the data is whitened. Whitening is a common preprocessing procedure, which makes the components of the data vectors mutually uncorrelated and normalizes their variances to unity. If principal component analysis is used for whitening, one can simultaneously filter out some additive noise. The whitened data matrix can be computed from the formula

$$\mathbf{Y} = \mathbf{\Lambda}_s^{-\frac{1}{2}} \mathbf{U}_s^T \mathbf{R} \quad (9)$$

where $\mathbf{\Lambda}_s$ is the diagonal matrix containing the $2K$ eigenvalues of the estimated data correlation matrix $\mathbf{RR}^T/N$, and $\mathbf{U}_s$ is the matrix containing the respective eigenvectors in the same order. The explanation for choosing $2K$ sources instead of $2KL$ is that the number of paths is usually impossible to determine, while the number of users can be found by uplink-downlink communication. The signal space is reduced to a dimension in which two sources correspond to each user's main transmission path.

After whitening, the FastICA algorithm [6] takes the following form, when the fourth-order statistics kurtosis is used:

1. Choose randomly an initial vector $\mathbf{w}(0)$ and normalize it to have a unit norm.
2. Compute the next estimate of a ICA basis vector after whitening using the fixed-point iteration rule

   $$\mathbf{w}(k) = \frac{1}{N}\mathbf{Y}[\mathbf{Y}^T\mathbf{w}(k-1)]^3 - 3\mathbf{w}(k-1)$$

   Normalize $\mathbf{w}(k)$ by dividing it by its norm.
3. Repeat step 2 until $\mid \mathbf{w}^T(k)\mathbf{w}(k-1) \mid$ is sufficiently close to 1.

In Step 2, $(\cdot)^3$ means elementwise operation. The above procedure is the fixed-point rule for estimating one ICA basis vector. The other ICA basis vectors can be estimated sequentially if necessary by projecting a new initial basis vector $\mathbf{w}(0)$ onto the subspace which is orthogonal to the subspace spanned by the previously found ICA basis vectors, and following then the same procedure; see [6, 5] for details.

4. SYNCHRONIZATION ALGORITHMS

Since in the CDMA downlink application only one signal, namely that of the desired user, is of interest, it would be desirable to estimate only one ICA basis vector corresponding to the desired user. To ensure that the first estimated ICA basis vector corresponds to the user of interest, we should have a good enough initial guess for $\mathbf{w}(0)$. This can be provided by using a training sequence:

$$\mathbf{w}(0) = \mathbf{Y}_P[b_{11}\dots b_{1P}]$$

Here the P first symbols of the user of interest are known, and $\mathbf{Y}_P$ is the corresponding part of $\mathbf{Y}$. The corresponding basis vector of the mixing matrix $\mathbf{G}$ in the original non-whitened space can then be found by applying the inverse transform

$$\mathbf{v} = \mathbf{U}_s\mathbf{\Lambda}_s^{-\frac{1}{2}}\mathbf{w} \tag{10}$$

This is in fact an estimate of the column of $\mathbf{G}$ corresponding to the desired user's mth symbol b_{1m}, that is

$$\mathbf{v} = \sum_{l=1}^{L} a_l\bar{\mathbf{g}}_{1l} \tag{11}$$

Let $l = 1$ be the index of the strongest path and let d_1 be the corresponding delay to be found. Full code vectors consist of bits $s_k \in \{1, -1\}$, and their 'late' part has the form given in Eq. (4). We take into account that the fading terms of the secondary paths are smaller in absolute value than for the main path. This assumption is valid if the signal power transmitted on those paths is smaller than that transmitted on the principal path. We also assume that the main path is the one which has the smallest chip delay $d_1 \leq d_l$, $l = 2, \dots, L$. Under these conditions the vector $\mathbf{v}$ will contain values absolutely close to zero, $\mid \mathbf{v}_i \mid \approx 0$, for $i = 1, \dots, d_1$. The delay of the main path d_1 will be thus given by the first index i for which $\mid \mathbf{v}_i \mid > t$, where t is a threshold depending on the strength of the other paths and the channel noise.

We have compared the procedure introduced above with a standard method called Constrained Minimum Output Energy (CMOE) method [4, 10]. CMOE minimizes the output energy of the interfering sources, leaving the desired source undistorted. Quite recently, CMOE has also been found to have a good synchronization capability [8]. The delay estimate obtained using the CMOE method is given by

$$\hat{d} = \arg\min_{d} \mathbf{g}(d)^T\hat{\mathbf{C}}_{xx}^{-1}\mathbf{g}(d) \tag{12}$$

Here

$$\mathbf{g}(d) = [s_1(C-d+1), \dots, s_1(C), s_1(1), \dots, s_1(C-d)]^T \tag{13}$$

and $\hat{\mathbf{C}}_{xx}$ is the estimate of the autocorrelation matrix of the observed data $\mathbf{R}$.

The simplest method for delay estimation is the matched filter [12, 14]. There the delay is found by the criterion

$$\hat{d} = \arg\max_{d} \mathbf{g}(d)^T\hat{\mathbf{C}}_{xx}\mathbf{g}(d) \tag{14}$$

5. SIMULATIONS

The algorithms were tested using gold codes of length $C = 31$. The number of users varied from $K = 2$ to $K = 8$, and the number of transmission paths was $L = 4$. The channel paths powers were -5, -5, -5, and 0 dB respectively for every user, and the signal-to-noise ratio (SNR) varied from 30 dB to 0 dB with respect to the main path. Thus the influence of secondary paths as well as the interference from the other users is quite significant.

The observation interval was $N = 300$. Fifty simulations were done for every number of users and SNR value. Our method is semi-blind because no parameters (codes, delays) need to be known in advance. The length of the training sequence for assuring convergence to the desired user in the fixed-point algorithm was only $P = 20$ symbols. The same number of input samples was used in each of the three estimation algorithms included in our study.

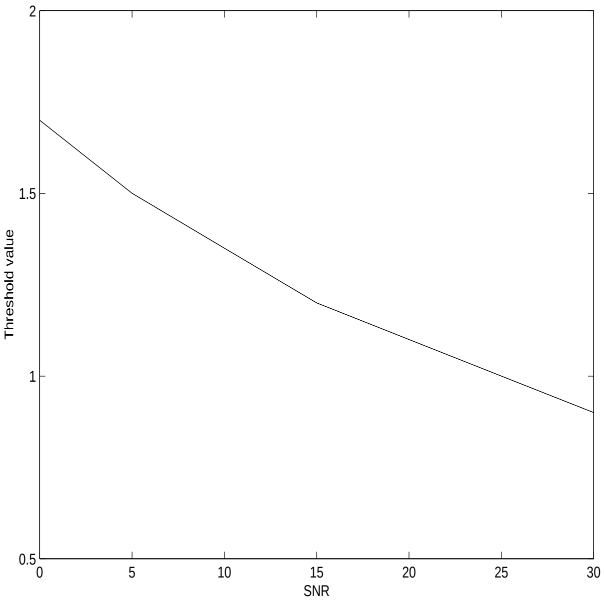

Figure 1: Optimum threshold values for different signal-to-noise ratios.

The synchronization given by the proposed method is conditioned by the convergence of the fixed-point algorithm. Moreover, the choice of the threshold is affected by the level of background noise and interfering other users. Figure 1 shows optimal values for the threshold as the function of the signal-to-noise ratio. These values were used in later simulations. Figures 2,3, and 4 show the achieved probability of acquisition using the FastICA, CMOE, and matched filter methods, respectively. Note that the scaling of the vertical axis is different in these figures. The figures show that the Independent Component Analysis approach based on the FastICA method provides the best estimation cabability.

6. CONCLUSIONS

In this paper, we have introduced a new method for timing acquisition in downlink CDMA communications. The method is based on a fast blind source separation (independent component analysis) algorithm. It performs better than conventional algorithms in a challenging environment. It has also the advantage of requiring a few training symbols only, which implies that it does not waste too much bandwidth. Our future research topic is to try to relax the need of training

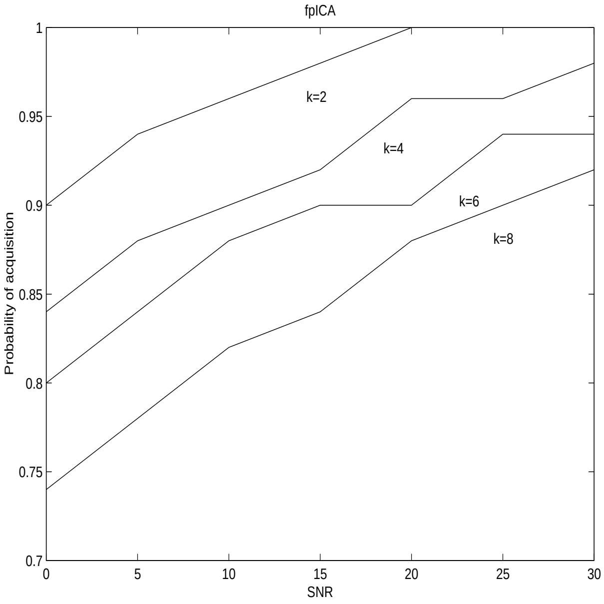

Figure 2: The mean probability of acquisition as a function of signal-to-noise ratio for the FastICA based method. Different curves corresponds to different numbers of users K = 2, 4, 6, and 8.

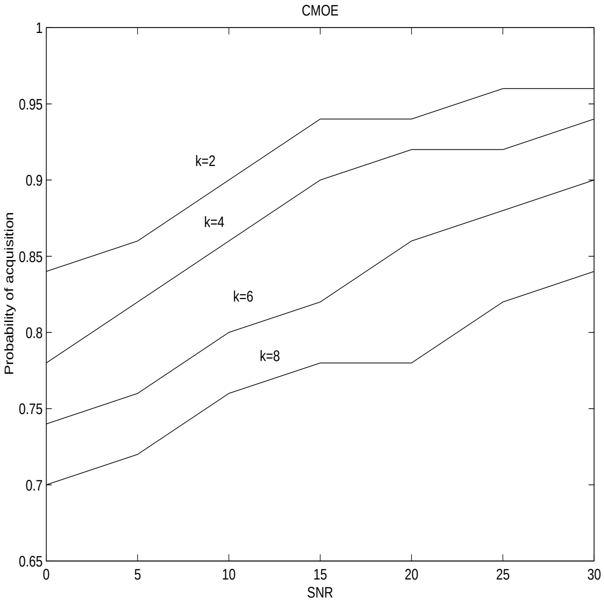

Figure 3: The mean probability of acquisition as a function of SNR for the CMOE method. Different curves corresponds to different number of users K = 2, 4, 6, and 8.

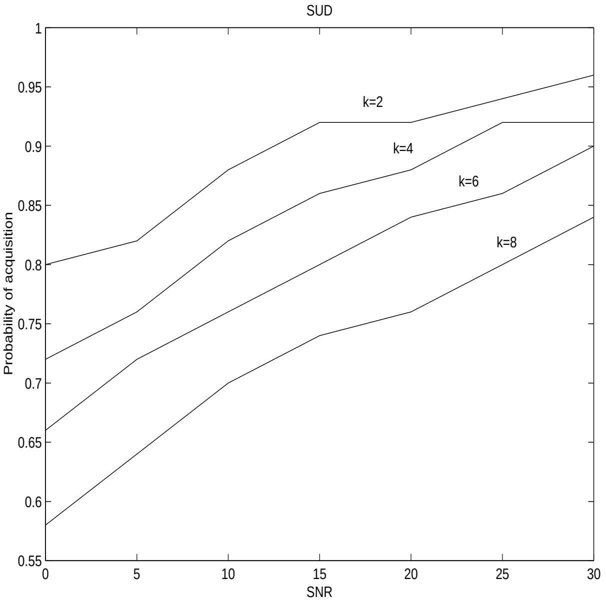

Figure 4: The mean probability of acquisition as a function of SNR for the matched filter. Different curves corresponds to different number of users K = 2, 4, 6, and 8.

symbols, and utilize more effectively the known code information.

Acknowledgment

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