BLIND DISCRIMINATION OF THE COHERENT OPTICAL SOURCES BY USING RETICLE BASED OPTICAL TRACKERS IS A NONLINEAR ICA PROBLEM

Ivica Kopriva
Institute for Defense Studies, R&D
Bijenička 46, 10000 Zagreb, Croatia
e-mail: ivica.kopriva@public.srce.hr

Harold Szu
ONR Code 313, 800 N
Quincy Arlington Virginia 22217-5660, USA
e-mail: szuh@onr.navy.mil

ABSTRACT

Reticle systems are considered to be the classical approach for estimating the position of a target in a considered field of view and are widely used in IR seekers. However, the disadvantage of the reticle trackers has been overly sensitivity to man-made clutters. We show that the nonlinear coherent version of Independent Component Analysis (ICA) theory can perhaps help alleviate the problem. When the processing device is redesigned properly by a band-pass filtering, the output signals are linear convolutive of the reticle transmission functions (rtf) considered as the unknown input source $s_i(t)$ signals in the context of ICA. That enables ICA neural network to be applied on the optical tracker output signals $x_i(t)$ giving on its outputs recovered rtf $s_i(t)$. Position of each optical source is obtained by applying appropriate demodulation method on the recovered source signals. The contribution of this paper is demonstrating that the coherence between optical sources results in a nonlinear ICA problem that becomes linearized, when the optical fields are incoherent, or, when the proper design of the optical tracker converts the nonlinear coherent model into linear one by band-pass filtering operation. Consequently, the multisource limitation of the reticle based optical trackers can in principle be overcome for both coherent and incoherent optical sources. We therefore conclude that requirements necessary for the ICA theory to work are fulfilled for both coherent and incoherent optical sources.

1. INTRODUCTION

The advantage of the reticle seekers (Fig. 1) is, because few detectors are used, simplicity and low cost, [1,2,5,6]. Owing to a spatial filtering effect of the reticle, the IR reticle tracker may exclude unwanted background signals, [1-3]. However, the major drawback of the reticle based trackers has been proven to be overly sensitivity on the IR countermeasures such as flares and jammers, [4-6]. It has been shown in [7,8] that an optical system based on a nutating reticle can be modified to resolve the multisource limitation problem, [4], by the combined use of ICA theory and appropriate modification of the optical tracker design, (see Fig. 4). Since reticle based optical systems are not widely understood, we present in Section 2 a brief description of the spatial filtering theory while more details can be found in [1-6, 9-12]. It was assumed in [7,8] that the intensity of the incident optical field is the sum of single intensities that results in a linear convolutive signal model. We present in Section 3 a more rigorous statistical optics based derivation of the signal model, [13,14]. It is shown that in a case of either partially or totally coherent optical fields the resulting signal model is in principle nonlinear, and is reduced in the limit of incoherence, is assumed a linear model is obtained. It is shown at the end of Section 3 how, by the proper design of the optical tracking system, it can be ensured that nonlinear signal model be transformed into linear one by simple linear band-pass filtering operation. In Section 4 a brief discussion of the ICA theory requirements is given for linear and nonlinear signal models. Simulation results are presented in Section 5. Conclusions are given in Section 6.

2. A BRIEF THEORY OF SPATIAL FILTERING

The reticle system performing modulation of the incident light flux provides the directional information for tracking passive arrivals and also suppresses unwanted background signals [1-3]. According to this type of the reticle and the relative motion produced by the scan pattern, the encoding method of the reticle may be classified into AM, FM and pulse code modulation. In addition, according to how the relative motion between the reticle and the optical spot is obtained we may classify reticle systems into fixed or moving reticle. When the reticle is fixed, a relative motion can be obtained by using rotating mirror which causes the light beam and hence the spot to either nutate or rotate in relation to the fixed reticle. In the opposite case, the spot forming optics is fixed while reticle performs either nutation or spinning. The general case of one moving reticle system is illustrated with Fig. 1. Moving reticle is placed in the focal plane of the collecting optics, while filed optics collects modulated light and focuses it on detector. The selective amplifier center frequency is usually the number of spoke pairs times the nutation or spinning frequency. The rising-sun reticle that is very often used in the nutating FM reticle trackers, [1,2,7,11,12], is shown on Fig. 2. In a case of either spinning or nutating reticle detector output voltage is proportional to the light irradiance behind the reticle according to [9,12]:

$$I(t)= I_P\int_0^R\int_{-\pi}^{\pi} T(r,\theta)\delta\left[r - r_0, \theta - (\Omega t - \theta_0)\right] r d\theta dr \qquad (1)$$

where $T(r,\theta)$ is rtf and r and θ are spatial variables of the rtf ranging from 0 to R and $-\pi$ to π, respectively. Also let the reticle nutation or spinning rate be Ω in rads^{-1} and let r_0 and θ_0 be the spatial coordinates of a point source that is imaged onto the reticle.

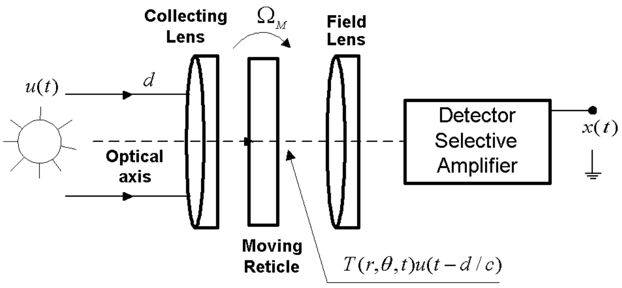

Figure 1: Optical tracker basic construction

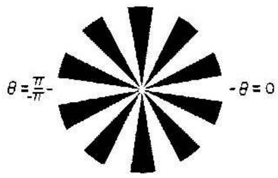

Figure 2. The rising-sun reticle

I_P in (1) is the peak irradiance of the point source through the rtf. Since the convolution of any function with delta function is the function located at the delta function coordinates the Eq. (1) becomes:

$$I(t) = I_P T(r_0, \Omega t - \theta_0) \qquad (2)$$

Therefore, the temporal response of all the subsequent spatial rtf is found replacing r with r_0 and θ with Ωt-θ_0. In optical trackers that generate FM signal by means of the rising-sun reticle, Fig. 2, and nutation the rtf is shown to be of the form [1,2,7,11,12]:

$$T(r,\theta,t) = I_p \cos[m\Omega t - m(r/a)\sin(\theta)] \qquad (3)$$

The optical spot performs circular motion around the center with coordinates (r_0,θ_0) relative to the center of the reticle. Necessary condition for Eq. (3) to hold is $(r_0/a)^2 << 1$. m in Eq. (3) is the number of spoke pairs of the reticle. Eq. (3) represents canonical form of the FM signal, [25], where frequency deviation from the carrier frequency is directly proportional with the spot r_0 coordinate. So by using nutating rising-sun reticle, both directional information, distance and azimuth, are encoded in the rtf. Instead of using nutation the relative motion between the spot and the reticle can be obtained by simple rotation or spinning, [9,10]. A number of spokes geometry is proposed for such purpose, [9,10]. It has been shown in [9] that rtf of the spinning FM reticle can be written in general form as:

$$T(r,\theta) = \frac{1}{2} + \frac{1}{2}\cos\left[m(r)\int_0^{\theta+\rho(r)} f(\alpha)d\alpha\right] \qquad (4)$$

The ½ DC term in Eq. (4) allows an average reticle transmission of ½ rather than zero (i.e. no light passing the reticle). Spinning FM reticles can be completely described by these three parameters: frequency vs. angle $f(\theta)$, frequency vs. radius $m(r)$ and phase or spoke function $\rho(r)$. The AM reticles are used in IR missile seekers in surface-to-air and air-to-air environments. It has been shown in [10] that it is possible to describe spinning AM reticles using three amplitude parameters (similar to previously described FM parameters): amplitude vs. angle $f(\theta)$, amplitude vs. radius $g(r)$, and phase $\rho(r)$. The general form of the spinning AM rtf that encodes both radial and angle coordinates is given by:

$$T(r,\theta) = \frac{1}{2} + Vg(r)[1 + mf(\theta)]\cos(k\theta) \qquad (5)$$

where V is a constant, m is the modulation index, $f(\theta)$ is the low frequency modulation signal and k is the carrier frequency that corresponds with the number of spoke pairs. Like in the FM reticle case the ½ DC term in Eq. (5) allows an average reticle transmission of ½ rather than zero (i.e., no light passing the reticle). A number of reticle geometry that generate AM signal which encodes angular, radial or phase information is given in [10].

3. STATISTICAL OPTICS BASED DERIVATION OF THE SIGNAL MODEL

We start with the problem of detecting optical radiation at point D when optical fields are emitted from two sources at points P_1 and P_2 (see Fig. 3). The optical field at point D is given as the sum of the individual fields multiplied by rtf:

$$u_D(t) = K_1 u_1(t)s_1(t) + K_2 u_2(t - \Delta t)s_2(t) \qquad (6)$$

where Δt represents relative time delay between u_1 and u_2 due to the path length difference i.e. $\Delta t = (d_2 - d_1)/c$. In order to be consistent with the ICA theory notation the ret rtf $T(r,\theta, t)$ will be replaced by $s(t)$ and called the source signal. Target coordinates r and θ will mostly be dropped in order to simplify notation. K_1 and K_2 are in general case complex constants representing path losses. We will assume here that they are real numbers. Detector will sense intensity obtained as [13,14]:

$$I_D(kT) = \left\langle u_D^*(t)u_D(t)\right\rangle \qquad (7)$$

where T represents detector averaging time and kT is new discretized time that allows treatment of nonstationarity.

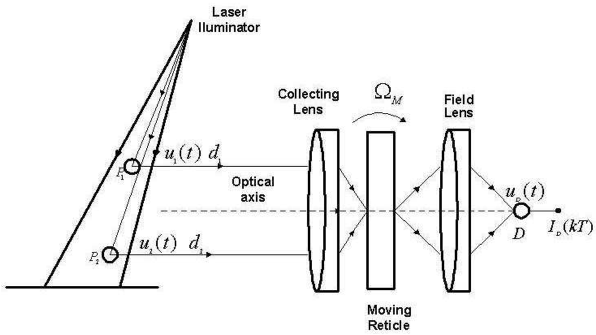

Figure 3. Generation of the coherent radiation

When (6) is applied to (7), I_D is obtained as:

$$I_D = I_1 s_1 + I_2 s_2 + 2K_1K_2\sqrt{I_1 I_2}\,\mathrm{Re}\{\gamma_{12}(\Delta t)\}s_1 s_2 \qquad (8)$$

In the Eq. (8) the time index is dropped in order to simplify notation. $\gamma_{12}(\Delta t)$ is the normalized mutual coherence function [13]:

$$\gamma_{12}(\Delta t) = \frac{\left\langle u_1(t)u_2^*(t-\Delta t)\right\rangle}{\sqrt{I_1 I_2}} \qquad (9)$$

Modulating functions s_1 and s_2 are functions of the coordinates of the corresponding optical sources only and are mutually independent in the sense of a factorized probability density function. They are also independent relative to the optical fields u_1 and u_2. It is therefore possible to write:

$$K_1^2\left\langle |u_1(t)|^2 s_1^2(t)\right\rangle = K_1^2\left\langle |u_1(t)|^2\right\rangle\left\langle s_1^2\right\rangle = I_1 s_1 ,$$

since $s_1^2 = s_1$ and $\langle s_1\rangle = s_1$ because the detector averaging process is fast in relation to the modulating function s_1. The same reasoning applies for s_2, which explains how the first two parts in Eq. (8) are obtained. The third part is obtained from:

$$2K_1K_2\left\langle u_1(t)u_2^*(t-\Delta t)s_1(t)s_2(t)\right\rangle =$$
$$2K_1K_2\left\langle u_1(t)u_2^*(t-\Delta t)\right\rangle\left\langle s_1(t)\right\rangle\left\langle s_2(t)\right\rangle$$

because s_1 and s_2 are independent of u_1 and u_2 and also mutually independent. Taking into account (9) and $\langle s_1\rangle = s_1$, $\langle s_2\rangle = s_2$, the third part of Eq. (8) is obtained. The photocurrent is obtained when the intensity I_D is expressed in terms of spectral irradiance and when detector spectral responsivity is taken into consideration, giving:

$$\begin{aligned} i(kT) = {} & A\int I_1(\lambda,kT)R(\lambda)d\lambda \times s_1(kT) \\ & + A\int I_2(\lambda,kT)R(\lambda)d\lambda \times s_2(kT) \\ & + 2K_1K_2\sqrt{\int I_1(\lambda,kT)R(\lambda)d\lambda \times \int I_2(\lambda,kT)R(\lambda)d\lambda} \\ & \times \mathrm{Re}\{\gamma_{12}(\Delta t)\}\times s_1(kT)s_2(kT) \end{aligned} \qquad (10)$$

where A is the detector sensing area and λ is wavelength.

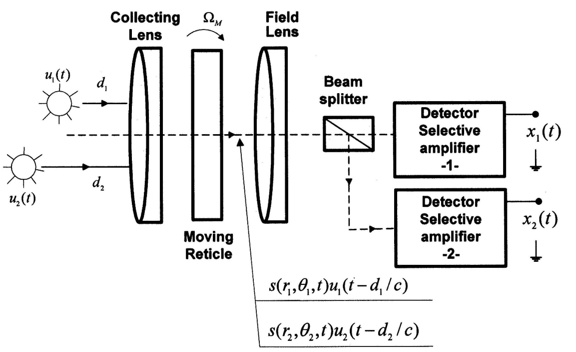

Figure 4. Modified optical tracker

When in accordance with Fig. 4 the beam splitter provides independent looks using two separate detectors, Eq. (10) can be used to obtain expressions for the corresponding photocurrents by inserting $\tau(\lambda)$ and $\rho(\lambda)$ into integrals over λ in Eq. (10). Here $\tau(\lambda)$ and $\rho(\lambda)$ are beam splitter transmission and reflection coefficients, respectively. Responsivity $R(\lambda)$ should be replaced with $R_1(\lambda)$ when i_1 is computed and with $R_2(\lambda)$ when i_2 is computed. The optical tracker output signals x_1 and x_2 are obtained as:

$$\begin{aligned} x_1(t) &= g_{11}(t)*s_1(t) + g_{12}(t)*s_2(t) + g_{13}(t)*[s_1(t)s_2(t)] \\ x_2(t) &= g_{21}(t)*s_1(t) + g_{22}(t)*s_2(t) + g_{23}(t)*[s_1(t)s_2(t)] \end{aligned} \qquad (11)$$

where impulse responses $g_{ij},\ i,j\in\{1,2,3\}$ can be identified from (8), (10) and (11) as:

$$\begin{aligned} & g_{11}(t) = A_1 g_1(t)B_{11}(\lambda,t), \quad g_{12}(t) = A_1 g_1(t)B_{12}(\lambda,t), \\ & g_{13}(t) = A_1 g_1(t)2K_1K_2\sqrt{B_{11}(\lambda,t)B_{12}(\lambda,t)}\times\mathrm{Re}\{\gamma_{12}(\Delta t)\} \\ & g_{21}(t) = A_2 g_2(t)B_{21}(\lambda,t), \ g_{22}(t) = A_2 g_2(t)B_{22}(\lambda,t), \\ & g_{23}(t) = A_2 g_2(t)2K_1K_2\sqrt{B_{21}(\lambda,t)B_{22}(\lambda,t)}\times\mathrm{Re}\{\gamma_{12}(\Delta t)\} \\ & B_{11}(\lambda,t) = \int \tau(\lambda)I_1(\lambda,t)R_1(\lambda)d\lambda , \\ & B_{12}(\lambda,t) = \int \tau(\lambda)I_2(\lambda,t)R_1(\lambda)d\lambda \\ & B_{21}(\lambda,t) = \int \rho(\lambda)I_1(\lambda,t)R_2(\lambda)d\lambda , \\ & B_{22}(\lambda,t) = \int \rho(\lambda)I_2(\lambda,t)R_2(\lambda)d\lambda \end{aligned} \qquad (12)$$

In relation to the signal model derived in [7] the model (11) is more complete. The linear model from [7] is obtained as a special case when incoherence between optical fields is assumed. If only the basic optical tracker construction is used (see Fig. 1) then Eq. (11) shows that optical tracker sees convolutive combination of the two modulating functions s_1 and s_2. It has been shown analytically in [4] that in such a case the optical tracker follows the centroid the coordinates of which are functions of the effective brightness of the two sources. The point is that optical tracker fails to determine the accurate coordinates of either of the two sources. That is known as IR jamming problem. Although this problem, associated with the reticle based tracking systems, is very old there are still new attempts to design jamming resistant reticle seekers, [5,6]. Basically these attempts assume that jammers can be detected on the basis of the energy and spectral discrimination. It is also assumed that, when jamming is detected, sensor signal is replaced with the predicted version based on the past values provided that the unknown target performs no maneuvering. A new approach was proposed in [7] and is extended for potential coherent illumination here. It is based on the independent component analysis theory and an appropriate modification of the optical tracker design that for the case of two sources is shown on Fig. 4. Generally, ICA enables source signals s_1 and s_2, Eq. (11), to be recovered on the basis of the observed signals x_1 and x_2 only. Since the nonlinear ICA algorithms, [15,23,24], are designed for the special types of the nonlinearity only, transformation of the nonlinear convolutive model (11) into linear one, [15-22], would be of great importance. The nonlinear ICA model (11) generated by the coherent optical sources has much more than the academic interest. Such situation happens when, according to Fig. 3, a laser illuminates two objects simultaneously for a passive detection mechanism without the range information. Let the optical tracking system is designed such that:

$$\omega_{\min} > \frac{\omega_{\max}}{2} \qquad (13)$$

where ω_{min} and ω_{max} are minimal and maximal corner frequencies of the optical tracker selective amplifiers, respectively. When the source signals s_1 and s_2 are multiplied two new parts of the frequency spectrum are generated that in general case cause nonlinear signal distortions. That can be avoided provided that the following inequalities are fulfilled:

$$\omega_{ij} + \omega_{ji} > \omega_{\max} \qquad (14)$$

$$|\omega_{ij} - \omega_{ji}| < \omega_{\min} \qquad (15)$$

where $\omega_{ij},\ i,j \in \{1,2\}$ are corner frequencies of the source signals s_1 and s_2. It is easy to show that this can be fulfilled if the optical tracking system is designed such that (13) is ensured. Then, the nonlinear model (11) can be transformed into linear one by applying linear band-pass filtering operation on the measured signals x_1 and x_2 of the signal model (11). The new model is obtained:

$$\begin{aligned}\hat{x}_1(t) &= \hat{g}_{11}(t) * s_1(t) + \hat{g}_{12}(t) * s_2(t)\\ \hat{x}_2(t) &= \hat{g}_{21}(t) * s_1(t) + \hat{g}_{22}(t) * s_2(t)\end{aligned} \tag{16}$$

where $\hat{g}_{ij}(t) = h_{BP}(t) * g_{ij}(t),\ i,j \in \{1,2\}$ and h_{BP} is impulse response of the linear band-pass filter with the corner frequencies ω_{min} and ω_{max}. By using online ICA algorithms it should be possible to recover the source signals $s_1(r,\varphi,t)$ and $s_2(r,\varphi,t)$ on the basis of the observed signals $x_1(t)$ and $x_2(t)$ only.

4. INTERPRETATION OF THE ICA THEORY REQUIREMENTS

There are three fundamental assumptions on which all ICA algorithms are based: *statistical independence of the source signals, non-Gaussianity of the source signals except one and non-singularity (or over-determinant case of more measurements than sources) of the mixing matrix in the model of the observed signals*. Here, it will be briefly examined whether these assumptions are fulfilled for the model of the modified optical tracker output signals (11) and (16). The statistical independence assumption of the source signals $s_1(r,\theta,t)$ and $s_2(r,\theta,t)$ is reasonable since they are generated by the two different (independent bodies) optical sources. Figures 5 and 6 show power spectrums of the two FM source signals Eq. (3). The absolute extreme values of the auto-correlation C_2s_1, C_2s_2 and cross-correlation $C_{11}s_1s_2$ as well as of the fourth order cumulants C_4s_1, C_4s_2 and cross-cumulant $C_{22}s_1s_2$ are given in table 1. It can be seen that in both cases the cross-statistics are approximately 10 times smaller than auto-statistics.

Table 1. Second and fourth order (cross-)statistics

Statistics	C_2s_1	C_2s_2	$C_{11}s_1s_2$	C_4s_1	C_4s_2	$C_{22}s_1s_2$
Max. value	0.5	0.5	0.068	0.375	0.369	0.034

The second assumption, that the source signals are non-Gaussian is also fulfilled for the following reasons. It has been shown in Eq. (3)-(5) that source signals are either FM or AM signals. These types of signals, as most communication signals, belong to the sub-Gaussian class of signals having negative kurtosis. For all FM rtf Eq. (3) and (4) the estimation of the kurtosis gives the value of roughly –1.5. For AM reticles Eq. (5) the estimated kurtosis lies in the interval [-0.9, -0.15] depending on the relative speed of motion between the target and the reticle. The third assumption is the nonsingularity of the mixing matrix when the convolutive model (11) i.e. (16) is transformed into the frequency domain. It has been discussed in Eq. (13)-(15) that under proper condition of the optical tracker design the nonlinear model (11), generated by the coherent optical sources, can be transformed into linear model (16) by simple linear bandpass filtering operation. Therefore, we shall examine the nonsingularity requirement on the linear model (16).

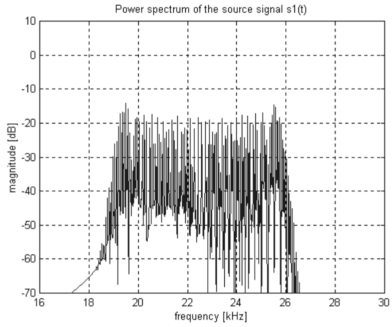

Figure 5. The source signal $s(r_1,\theta_1)$, Eq. (3).

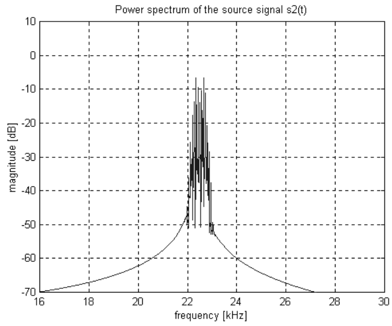

Figure 6. The source signal $s(r_2,\theta_2)$, Eq. (3).

The nonsingularity requirement means that measured signals $\hat{x}_1$ and $\hat{x}_2$ must be linearly independent combinations of the source signals s_1 and s_2, which ensures a benefit from using two sensors. It is shown in [7,26] that this assumption holds. The signal model (16) transformed into frequency domain yields:

$$\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} = \begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix} \times \begin{bmatrix}S_1\\ S_2\end{bmatrix} \tag{17}$$

where all quantities in the Eq. (17) are Discrete Fourier Transforms (DFTs) of the related time domain quantities in the signal model (16). The frequency variable ω is dropped in Eq. (17) in order to simplify notation. The nonsingularity condition is transformed into:

$$G_{11}G_{22} - G_{12}G_{21} \neq 0 \tag{18}$$

Provided that $R_1(\lambda) \cong R_2(\lambda) \cong R(\lambda)$ the inequality (18) is transformed in [7,8,26]:

$$\int_\lambda \frac{\tau(\lambda)R(\lambda)I_1(\lambda,t)}{\lambda}d\lambda \times \int_\lambda \frac{R(\lambda)I_2(\lambda,t)}{\lambda}d\lambda \neq \int_\lambda \frac{\tau(\lambda)R(\lambda)I_2(\lambda,t)}{\lambda}d\lambda \times \int_\lambda \frac{R(\lambda)I_1(\lambda,t)}{\lambda}d\lambda \tag{19}$$

Condition (19) and consequently condition (18) will be fulfilled when:

$$\tau(\lambda) \neq const. \tag{20}$$

over the wavelength region of interest that is fulfilled for the real beam splitters.

5. SIMULATION RESULTS

To reconstruct source signals the feedback separation network described with (21) is applied. Reasons for using the feedback network is to avoid whitening effect [18].

$$y_1(k) = x_1(k) - \sum_{i=1}^{M} w_{12}(i) y_2(k-i)$$
$$y_2(k) = x_2(k) - \sum_{i=1}^{M} w_{21}(i) y_1(k-i) \qquad (21)$$

To reconstruct modified optical tracker source signals an online separation algorithm based on the infomax criterion [16,18] will be used. The infomax learning rules with *z=g(y)=tanh(y)* for feedback separation network (21) are given with:

$$w_{ij}(k+1,l) = w_{ij}(k,l) + \mu 2 \tanh(y_i) y_j(k-l) \quad (22)$$

where in (22) $i, j \in \{1,2\}$ and $l = 1,2,\ldots,M$, and M is order of the cross-filters in the feedback separation network. Measured signals x_1 and x_2 were generated according to the nonlinear signal model (11) on the basis of the two FM source signals s_1 and s_2 the power spectrums of which are shown on Fig. 5 and 6. The total coherence case, $\gamma_{12}(\Delta t) = 1$, was assumed. Power spectrum of the measured signal x_1 is shown on Fig. 7.

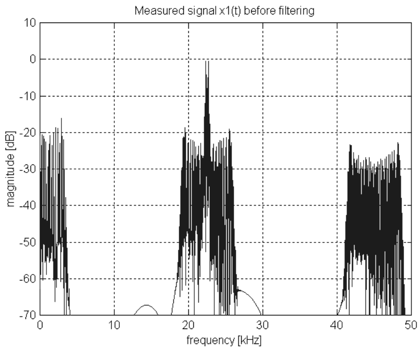

Figure 7. Power spectrum of x_1, Eq. (11).

The nonlinear effect due to the nonlinear part in the signal model (11) can be observed. Power spectrum of the measured signal x_2 looks very similarly. When, in accordance with the exposed analysis Eq. (12)-(15), the linear bandpass filtering is applied on the measured signals x_1 and x_2, signals $\hat{x}_1$ and $\hat{x}_2$ are obtained, Eq. (16). Power spectrum of the signal $\hat{x}_1$ is shown on Fig. 8.

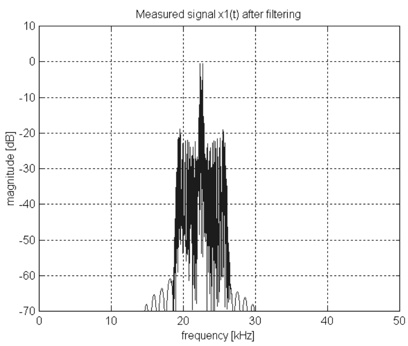

Figure 8. Power spectrum of $\hat{x}_1$, Eq. (11).

It is obvious that nonlinear effect has been eliminated. Power spectrum of the signal $\hat{x}_2$ looks very similarly. When FM demodulator is applied on either signal $\hat{x}_1$ or signal $\hat{x}_2$, only the IR optical source that was placed near the center of the filed of view (FOV) can be discriminated. If, however, the entropy based ICA algorithm, Eq. (22), is applied on the signals $\hat{x}_1$ and $\hat{x}_2$ the influence of the IR source placed near the center of the FOV can be eliminated and both IR sources can be discriminated. Power spectrums of the signals y_1 and y_2, according to Eq. (21), are shown on Fig. 9 and 10.

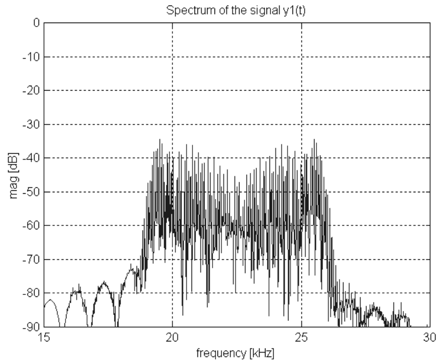

Figure 9. Power spectrum of y_1, Eq. (26).

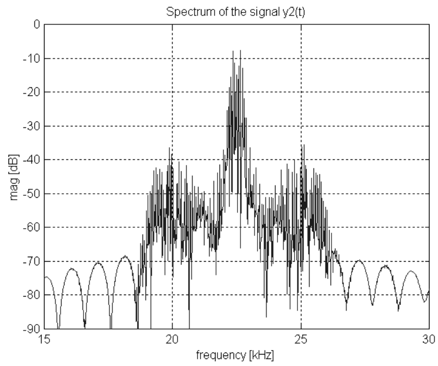

Figure 10. Power spectrum of y_2, Eq. (26).

It can be observed in signal y_1 that influence of the IR source placed near the center of the FOV is eliminated. Signal y_1 represents reconstructed version of the source signal s_1 while y_2 represents reconstructed version of the source signal s_2.

6. CONCLUSION

ICA approach to resolve the multisource limitation of the reticle based optical trackers is exposed in the paper. When redesigned adequately optical trackers produce output signals that are linear convolutive combinations of the rtf considered the source signals in the context of the ICA theory. Each function corresponds with single optical source position. That enables ICA neural network to be applied on the optical tracker output signals giving on its outputs recovered rtf. Position of each optical source is obtained by applying appropriate demodulation method on the recovered source signals. The three conditions necessary for the ICA theory to work (statistical independence and non-Gaussianity of the source signals and nonsingularity of the mixing matrix) are shown to be fulfilled in principle for any kind of the reticle geometry. A statistical optics based analysis is performed that yields a mathematical model of the output signals of a modified reticle based optical tracker. It has been shown that coherence between optical sources produces a nonlinear signal model that becomes linear when optical sources are incoherent. It has been shown additionally that by the proper optical tracker design the nonlinear model, generated by the coherent optical sources, can be converted into linear one by simple linear bandpass filtering operation. It has been also shown that the nonsingularity of the mixing matrix in the frequency domain can be ensured requiring the beam splitter transmission coefficient be non-constant over the wavelength region of interest. Thus the requirements necessary for the ICA theory to work are fulfilled for both coherent and incoherent optical sources. Consequently, the multisource limitation of the either nutating or spinning reticle based optical trackers can in principle be overcome for both coherent and incoherent optical sources. Early related work in case of more than two sources is reported in seven optical spectral measurements of Landsat imagery where a single pixel has been blindly de-mixed to discover a multiple ground radiation sources, [27].

7. REFERENCES

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