T-61.231 Principles of Pattern Recognition

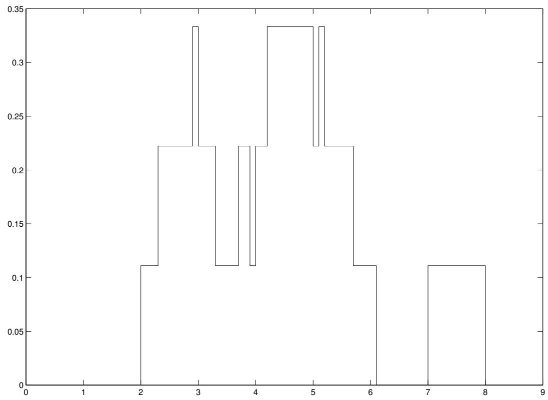
Answers to exercise 3: 7.10.2002

1. Parzen window $\Theta(x) = \Theta\left(\frac{\overline{x}-\overline{x^i}}{h_n}\right) = \begin{cases} 1, \text{ if } \overline{x} \text{ is in hypercube center } \overline{x^i}, \text{ side length } h_n \\ 0, \text{ otherwise} \end{cases}$

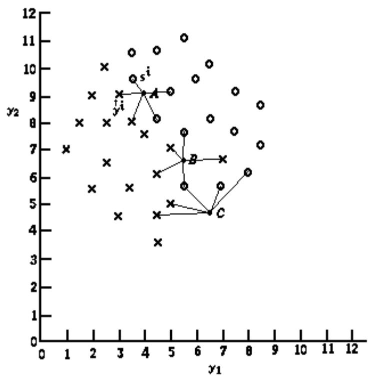
   $k_n = \sum_{i=1}^{n} \Theta\left(\frac{\overline{x}-\overline{x^i}}{h_n}\right)$ is the number of samples in the hypercube and n the total number of samples.

   $p_n(\overline{x}) = \frac{k_n/n}{V_n} = \frac{1}{nV_n}\sum_{i=1}^{n} \Theta\left(\frac{\overline{x}-\overline{x^i}}{h_n}\right)$, where V_n is the volume of the hypercube.

   Now $\overline{x^i} = y^{(i)}$, $h_n = 1$, $V_n = 1$ and $n = 9$. Thus



2. Point X is classified according to it's 5 nearest neighbors. The distance metric used is the Euclidean distance and the decision rule the voting result of the 5 nearest neighbors.



Point	Votes for O	Votes for X	Resulting class
A	3	2	O
B	2	3	X
C	3	2	O

3. **a)** $y^{(i)} = [y_1^{(i)}\ y_2^{(i)}]^T, s^{(j)} = [s_1^{(j)}\ s_2^{(j)}]^T$. A point $\hat{y} = [y_1\ y_2]^T$ is on the decision boundary, if

$$d(\hat{y}, y^{(i)}) = d(\hat{y}, s^{(j)})$$
$$\sqrt{(\hat{y}_1 - y_1^{(i)})^2 + (\hat{y}_2 - y_2^{(i)})^2} = \sqrt{(\hat{y}_1 - s_1^{(j)})^2 + (\hat{y}_2 - s_2^{(j)})^2}$$

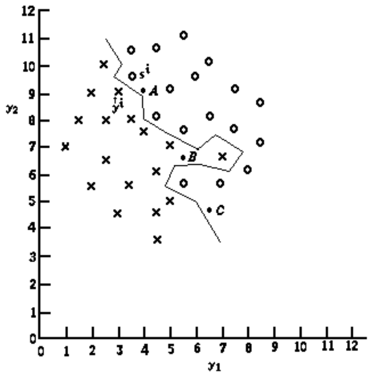
$$\Leftrightarrow \hat{y}_1 2(s_1^{(j)} - y_1^{(i)}) + \hat{y}_2 2(s_2^{(j)} - y_2^{(i)}) + (y_1^{(i)})^2 - (s_1^{(j)})^2 + (y_2^{(i)})^2 - (s_2^{(j)})^2 = 0$$

and by setting $a = 2(s_1^{(j)} - y_1^{(i)})$, $b = 2(s_2^{(j)} - y_2^{(i)})$ and $c = (y_1^{(i)})^2 - (s_1^{(j)})^2 + (y_2^{(i)})^2 - (s_2^{(j)})^2$ it is clear that the resulting equation $a\hat{y}_1 + b\hat{y}_2 + c = 0$ represents a line.

By taking the points from the figure,

$$\begin{cases} s_1^{(j)} = 3.6 \\ s_2^{(j)} = 9.5 \\ y_1^{(i)} = 3.0 \\ y_2^{(i)} = 9.0 \end{cases} \Rightarrow \begin{cases} a = 1.2 \\ b = 1.0 \\ c = -13.12 \end{cases} \Rightarrow \hat{y}_2 = -1.2\hat{y}_1 + 13.12$$

b)



c)