T-61.231 Principles of Pattern Recognition

Exercise 5: 21.10.2002

1. Means of two classes are found to be $\underline{m_1} = (\ -2 \quad -2\ )^T$ and $\underline{m_2} = (\ 2 \quad 2\ )^T$. Inner-class scatter matrices are (α is a parameter)

$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{S}_2 = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix}.$$

   a) Draw the density functions of the classes by assuming classes to be (approximately) Gaussian distributed.

   b) Calculate the direction of Fisher's linear discriminant by using the formula $\underline{\hat{w}} = S_W^{-1}(\underline{m_1} - \underline{m_2})$ (equation 4-12 in the book by Schalkoff).

   c) Find the direction of Fisher's linear discriminant by calculating eigenvectors for matrix $S_\omega^{-1} S_B$.

   d) Sketch Fisher's linear discriminant, when $\alpha = 1, \alpha = 4, \alpha \to \infty, \alpha \to 0$. Does the result look reasonable?

2. Let's choose the nominator of the equation (4-6b) in Schalkoff to be a criterion function for Fisher's linear discriminant. That means we neglect the variance of the projected data inside the class. Our criterion function is then $J(\omega) = (\overline{m}_1 - \overline{m}_2)^2$. Maximize $J(\omega)$.

3. Prove that given $O(N, l)$ is the number of groupings that can be formed by $(l-1)$ dimensional hyperplanes to separate the N points into two classes, taking all possible combinations

$$O(N, l) = 2 \sum_{i=0}^{l} \binom{N-1}{i}$$