T-61.231 Principles of Pattern Recognition

Exercise 2: 30.9.2002

1. The pattern space is 1-dimensional (the x axis) and the densities of two classes are

$$p(x|\omega_1) = 0.5e^{-|x-m_1|}, p(x|\omega_2) = e^{-2|x-m_2|}$$

   a) Let $m_1 = 0, m_2 = 2$ and the decision regions $R_1 = \{x|x \leq 1\}$ and $R_2 = \{x|x > 1\}$. Compute the probabilities of error ϵ_1 and ϵ_2. Plot a figure.

   b) How should one place the border between R_1 and R_2, in order that $\epsilon_1 = \epsilon_2$?

2. Assume now that the ratio of the costs of incorrest decisions for the class densities given above (exercise 1) is $(\lambda_{21} - \lambda_{22})/(\lambda_{12} - \lambda_{11}) = \frac{1}{2}$. Compute the borders of the region R_2 as functions of the *a priori* probability p. Find those values of p, for which R_2 is reduced to an empty region. We assume here that the classifier is Bayesian.

3. Take two densities $p(x|\omega_1)$ and $p(x|\omega_2)$ to be Gaussian densities of a scalar variable with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 0.5$ and equal means $(= 0)$. Assume that the other parameters are $P(\omega_1) = 0.7$ and $(\lambda_{21} - \lambda_{22})/(\lambda_{12} - \lambda_{11}) = 7$. How do you classify the following patterns: $\hat{x}_1 = 2.8, \hat{x}_2 = 0.2, \hat{x}_3 = 1.4, \hat{x}_4 = -0.6, \hat{x}_5 = -1.9$?

4. Assume a two-class Bayes classifier where the patterns in the two classes are Gaussian distributed with parameters

   a)

$$\mu_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mu_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \mathbf{\Sigma}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \mathbf{\Sigma}_2 = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}.$$

   b)

$$\mu_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \mu_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \mathbf{\Sigma}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \mathbf{\Sigma}_2 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}.$$

   For both cases, plot the decision surface and the approximative shapes of the densities $p(\mathbf{x}|\omega_1)$ and $p(\mathbf{x}|\omega_2)$. We may assume that $P(\omega_1) = P(\omega_2) = 1/2$ and $\lambda_{21} - \lambda_{11} = \lambda_{12} - \lambda_{22}$.

5. Let $x_1, x_2, \ldots, x_n$ be independent non-negative integers from Poisson distribution with expectation value $E[x] = \lambda$. This corresponds to a discrete distribution
$p(x|\lambda) = \lambda^x e^{-\lambda}/x!, x \geq 0$, when $E[x] = \mathrm{Var}[x] = \lambda$. Find the maximum likelihood estimate for the parameter λ. Is it unbiased?