Tik-61.231 Principles of Pattern Recognition

Exercise 10: 27.11.2000

1. Consider the following proximity matrix:

$$P = \begin{bmatrix} 0 & 4 & 9 & 6 & 5 \\ 4 & 0 & 1 & 8 & 7 \\ 9 & 1 & 0 & 3 & 2 \\ 6 & 8 & 3 & 0 & 1 \\ 5 & 7 & 2 & 1 & 0 \end{bmatrix}$$

   Apply the single and complete link algorithms to P and comment on the resulting dendreograms. *(Theodoridis, exercise 13.10, p. 437)*

2. The C-means clustering algorithm (or the LBG coding algorithm) can be used for the coding of images. What is the compression ratio if windows of $(k \times k)$ pixels sampled from a colored image (8 bits/color, 3 colors) are coded using 2^n code vectors.

3. Let us consider 1-dimensional self-organizing map (SOM), where both weights of neurons m_i and input vectors x are always in the unit circle. That means they are angles between $[0, 2\pi[$. There are 5 weights and their locations in the beginning of the learning are shown in Figure 1.

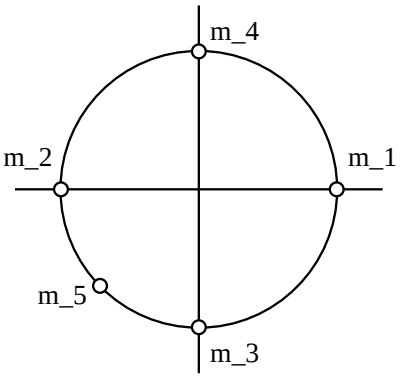

Figure 1:

   Learning rule (Kohonen's rule):

   1. Choose point x arbitrarily from the unit circle.
   2. Find among 5 weights the Best Matching Unit (BMU) m_c whose distance (angle) from the point x is the smallest.
   3. Update the weight m_c and its 2 nearest neighbors. Neighborhood is defined so that e.g. neighbors of m_3 are always (independent from the location) m_2 and m_4. Neighborhood is cyclical: neighbors of m_5 are m_4 and m_1, and similarily neighbors of m_1 are m_5 and m_2. Weights are updated so that they are moved along the shortest route to the point in the middle of the input x and their current location.

The problem is: you have to arrange the weights into the ascending or descending order in the unit circle by using the learning rule given above. You may choose inputs x arbitrarily. Try to minimize the number of learning steps.

(Hint: minimum solution has 2 steps, that means only 2 inputs x have to be chosen. Can you find out how?)

4. Assume the energy function

$$C = \sum_q \sum_r h(r-q) \sum_{x \in V_r} \|x - m_q\|^2$$

where r and q are map indeces, m_q are neuron weights, $h(r-q)$ is the space invariant neighborhood function, and V_r is the set of input points for which neuron r is the best-matching unit. It can be shown that this is an energy function for the Kohonen algorithm if the set of input points is finite.

a) Form the gradient of function C with respect to the weight vector m_q and let it be zero. This defines an equilibrium point for the net.

b) If we assume that all the sets V_r have an equal amount of N points, and the average of set V_r is denoted as

$$c_r = 1/N \sum_{x \in V_r} x,$$

show that in the equilibrium point defined by a) it holds

$$m_q = \sum_r h(q-r) c_r.$$

5. LVQ1-algorithm is applied for one-dimensional task in which the inputs are from two classes. The prior probabilities of the classes are equal. The value of input x is zero or one if it belongs to class ω_1 or ω_2, respectively. Each class is presented with a single weight, w_1 or w_2. Where do the weights converge? Consider different initial values for the weights.