# Tik-61.261 Principles of Neural Computing

Raivio, Venna

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1. An odd sigmoid function is defined by

$$\varphi(v) = \frac{1 - \exp(-av)}{1 + \exp(-av)} = \tanh(av/2),$$

where tanh denotes a hyperbolic tangent. The limiting values of this second sigmoid function are $-1$ and $+1$. Show that the derivate of $\varphi(v)$ with respect to $v$ is given by

$$\frac{d\varphi}{dv} = \frac{a}{2}(1 - \varphi^2(v)).$$

What is the value of this derivate at the origin? Suppose that the slope parameter $a$ is made infinitely large. What is the resulting form of $\varphi(v)$?

2. (a) Show that the McCulloch-Pitts formal model of a neuron may be approximated by a sigmoidal neuron (i.e., neuron using a sigmoid activation function with large synaptic weights).

   (b) Show that a linear neuron may be approximated by a sigmoidal neuron with small synaptic weights.

3. Construct a fully recurrent network with 5 neurons, but with no self-feedback.

4. Consider a multilayer feedforward network, all the neurons of which operate in their linear regions. Justify the statement that such a network is equivalent to a single-layer feedforward network.

5. (a) Figure 1(a) shows the signal-flow graph of a recurrent network made up of two neurons. Write the nonlinear difference equation that defines the evolution of $x_1(n)$ or that of $x_2(n)$. These two variables define the outputs of the top and bottom neurons, respectively. What is the order of this equation?

   (b) Figure 1(b) shows the signal-flow graph of a recurrent network consisting of two neurons with self-feedback. Write the coupled system of two first-order nonlinear difference equations that describe the operation of the system.

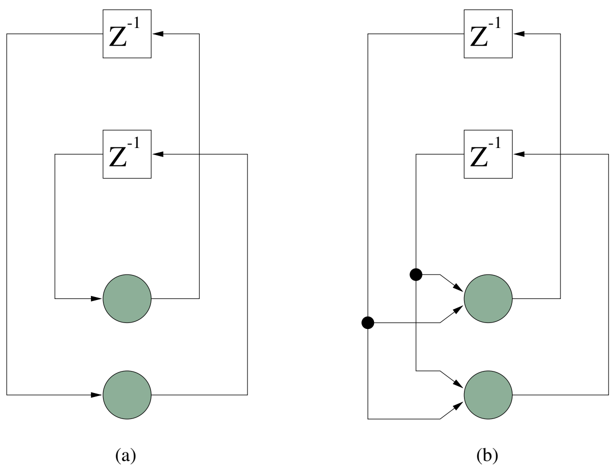


Figure 1: The signal-flow graphs of the two recurrent networks.