T-61.3010 Digitaalinen signaalinkäsittely ja suodatus

Mid term exam 1, Fri 12.3.2010 at 13-16, Hall A.

You are allowed to do MTE1 only once either 6.3. or 12.3.

You are not allowed to use any calculators or math reference books. A list of formulas is delivered in the exam. A special form is delivered for Problem 1.

Return a special form and the other answer paper separately. Both ones have to have at least student number and name written on. Problem paper and the formulas you may keep.

Problem 3 is a course feedback which is open from Sat 6-March to Mon 22-March 2010. However, questionnaire is only in Finnish, so the feedback from non-Finnish students is collected in our non-Finnish group meetings.

1) (0-9 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically. **BLACKEN THE BOX** of your choice.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 9 and the minimum 0.

   1.1 Consider a sequence $x[n] = A_1 \cos(\omega_1 n + \theta_1) + A_2 \cos(\omega_2 n + \theta_2) + A_3 \cos(\omega_3 n + \theta_3)$, where fundamental periods of each subsequence are $N_1 = 6$, $N_2 = 8$ and $N_3 = 10$, and A_i are non-zero. What can be said about periodicity of sequence $x[n]$?
   (A) Fundamental period N_0 exists if and only if all phases are zero: $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$
   (B) Fundamental period is greatest common divisor/factor (GCD), that is, $N_0 = 2$
   (C) Fundamental period is least common multiple (LCM), that is, $N_0 = 120$
   (D) Fundamental period is product of periods of subsequencies, that is, $N_0 = 480$

   1.2 Compute linear convolution $y[n] = h[n] \circledast x[n]$ of sequences $x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] = \{\underline{1},\ 2,\ 1\}$ and $h[n] = \delta[n] + 2\delta[n-2] = \{\underline{1},\ 0,\ -2\}$. where underline shows the origin.
   (A) Length of $y[n]$ is 5
   (B) $y[n] = 0$, when $n \leq 0$
   (C) $y[n] = 0$, when $n \geq 3$
   (D) $y[1] = 0$

   1.3 Two LTI systems $h_1[n]$ and $h_2[n]$ in parallel connection form the total impulse response $h[n]$ of the system. We know that $h_2[n] = \{1,\ \underline{2},\ -1\}$ and $h[n] = \{-2,\ -5,\ \underline{1},\ 3,\ -1\}$, where underline shows the origin. Hence, the unknown $h_1[n]$ is of form
   (A) $h_1[n] = a \cdot \delta[n+2] + b \cdot \delta[n+1] + c \cdot \delta[n] + d \cdot \delta[n-1] + e \cdot \delta[n-2]$
   (B) $h_1[n] = b \cdot \delta[n+1] + c \cdot \delta[n] + d \cdot \delta[n-1]$
   (C) $h_1[n] = d \cdot \delta[n-1] + e \cdot \delta[n-2] + f \cdot \delta[n-3]$
   (D) $h_1[n]$ is a causal filter
   where $\{a,\ b,\ c,\ d,\ e,\ f\} \in \mathbb{R}$ and non-zero.

   1.4 Two-point moving average filter:
   (A) It is FIR
   (B) It can have linear phase response
   (C) The order of the filter is 1
   (D) The structure of the filter contains at least one feedback loop

   1.5 We know a band-limited spectrum $|X(j\Omega)|$ of an analog real-valued signal $x(t)$, see Figure 1(a). The signal is sampled with sampling frequency $f_T = 10000$ Hz.
   (A) The spectrum $|X(e^{j\omega})|$ of the sampled sequence in range $[0, f_T/2]$ is in Figure 2(a). (y-axis values propotional.)
   (B) The spectrum $|X(e^{j\omega})|$ of the sampled sequence in range $[0, f_T/2]$ is in Figure 2(b). (y-axis values propotional.)
   (C) The obtained sequence $x[n]$ is a sinusoidal of form $x[n] = \cos(\omega_0 n + \theta)$, where $\omega_0 = 2\pi(f_0/f_T)$ is normalized fundamental angular frequency
   (D) All those frequency components, whose period T_i is longer than $2/f_T$ seconds, alias to lower frequencies in range $[0, f_T/2]$ Hz of the digital spectrum $|X(e^{j\omega})|$, and therefore cannot be recovered back in the ideal D/A reconstruction

1.6 Fourth order LTI filter has poles at $p_1 = a$, $p_2 = -a$, $p_3 = bj$, and $p_4 = -bj$, where a and b are real-valued and $0 < a < b < 1$. All zeros are in the origin. Which of the following can be the magnitude response of the filter?

(A) Figure 3(a)
(B) Figure 3(b)
(C) Figure 3(c)
(D) Figure 3(d)

1.7 The impulse response of a LTI filter is

$$h[n] = 4 \cdot (-0.8)^n \mu[n] - 3 \cdot (-0.6)^n \mu[n]$$

(A) Order of the filter is 1
(B) Zeros are at $z_1 = 0.8$ and $z_2 = 0.6$
(C) It has a linear phase response
(D) It is a highpass filter

1.8 Discrete Fourier transform (DFT) of a sequence $x_1[n] = \{\underline{2}, 1, 2, 1\}$ is

$$X_1[k] = \sum_{n=0}^{3} x_1[n] W_N^{nk} = \{\underline{6},\ 0,\ 2,\ 0\}$$

and correspondingly for $x_2[n]$ there are $x_2[n] = \{\underline{1},\ 2,\ 3,\ 4\}$ and $X_2[k] = \{\underline{10},\ -2+2j,\ -2,\ -2-2j\}$. Compute DFT $X_3[k]$ of a sequence $x_3[n] = 2x_1[n] - x_2[n]$. (DFT can be found in formula table.)

	$k =$	0	1	2	3
(A)	$X_3[k] =$	2	$2-2j$	6	$2+2j$
(B)	$X_3[k] =$	3	0	-1	$2j$
(C)	$X_3[k] =$	10	$2-3j$	4	$3-2j$
(D)	$X_3[k] =$	22	$-2+2j$	2	$-2-2j$

1.9 A simple lowpass filter is given with the frequency response

$$H_{LP}(e^{j\omega}) = \frac{1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}}{4}$$

whose magnitude response is in Figure 4(a). Using a frequency shift receive easily a simple highpass filter, whose magnitude response is in Figure 4(b). Compute the impulse response for the highpass filter. Hint: formula table.

(A) $h_{HP}[n] = 4 \cdot \big(\delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3]\big)$
(B) $h_{HP}[n] = (-0.25) \cdot \big(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\big)$
(C) $h_{HP}[n] = 0.25 \cdot \big(\delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]\big)$
(D) $h_{HP}[n] = e^{-j1.5\omega n} \cdot \big(\cos(0.5\omega n) + \cos(1.5\omega n)\big)$

1.10 In Matlab we are computing output of a LTI system as follows:

```
x = [9 8 9 9 8 1 2 3 2 2 1 9 8 7 9 8];  % input
y = zeros(size(x));                     % initialize output with zeros

for k = [2 : length(x)-1]
  y(k) = x(k) - x(k+1) - 1.1*y(k-1);
end;
```

What can be said about properties or action of the corresponding LTI system?

(A) It is FIR
(B) Output values `y` grow to infinitely large so that the program stops
(C) It is not a causal filter
(D) Group delay is $\tau(\omega) = -0.5$

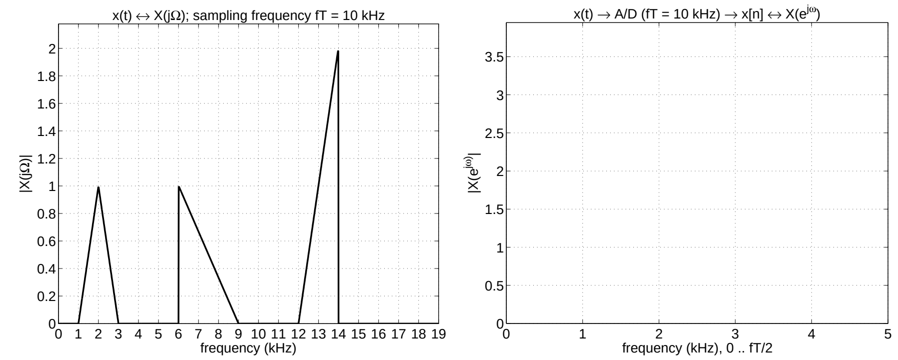

Figure 1: Statement 1.5: (a) Spectrum $|X(j\Omega)|$ of analog signal $x(t)$, (b) empty axis $f \in [0, f_T/2]$ for sketching.

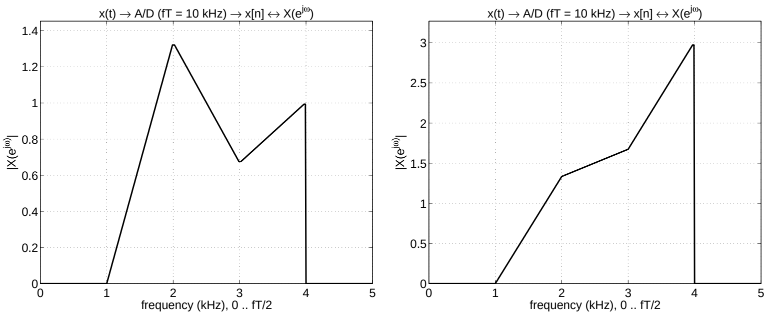

Figure 2: Statement 1.5: (a) option **(A)** , (b) option **(B)** .

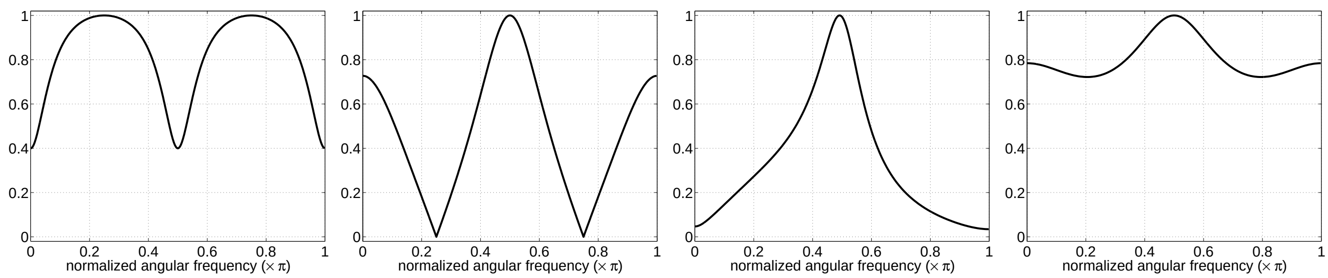

Figure 3: Statement 1.6, options (a) **(A)** , (b) **(B)** , (c) **(C)** , (d) **(D)** .

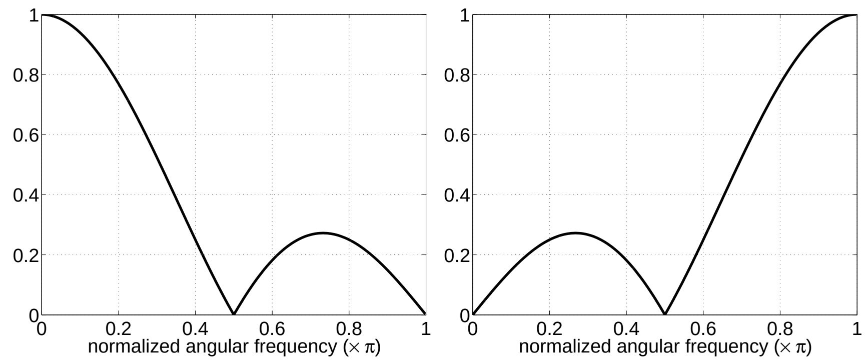

Figure 4: Statement 1.9: (a) Original lowpass filter $|H_{LP}(e^{j\omega})|$, (b) Desired highpass filter $|H_{HP}(e^{j\omega})|$.

2) (6 p) Consider a discrete-time linear and time-invariant system, whose transfer function is

$$H(z) = \frac{1}{1 + 0.2z^{-1}} + \frac{1 + 0.2z^{-1}}{1 - 0.8z^{-1}}, \quad |z| > 0.8$$

Examine the filter and its behavior with tools given in the course. Write down the facts as clearly as possible.

3) (1 p) Course feedback in non-Finnish group meetings.