T-61.3010 Digital Signal Processing and Filtering

1st mid term exam, March Sat 10th 2007 at 10-13. Main building. (UPDATED)

You are allowed to take 1st MTE only once either Mar 6th or Mar 10th.

You are not allowed to use any calculators or math books of your own. A formula table and a form for Problem 1 are delivered.

All concept papers have to be returned, **the form from Problem 1 separately.** You can keep the question paper and the formula paper.

Write down all **necessary steps** in your solution of Problem 2.

1) (14 x 1p, max 12 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

In these statements the region of convergence (ROC) in z-transforms is considered to be the area outside of the outmost pole.

1.1 The frequency response $H(e^{j\omega})$ of LTI system
- **(A)** is always real-valued for real-valued impulse response $h[n]$
- **(B)** is periodic with 2π (normalized angular frequency)
- **(C)** can be represented as a weighted sum of amplitude response and phase response
- **(D)** shows which frequency components (sinusoidals) the input sequence contains

1.2 Examine the sequence $x[n] = \cos((\pi/8)n) + \cos((\pi/12)n)$. What can be said of the periodicity of $x[n]$?
- **(A)** The fundamental period is $N_0 = 8$
- **(B)** The fundamental period is $N_0 = 12$
- **(C)** The fundamental period is $N_0 = 24$
- **(D)** None of above is true

1.3 Examine a complex-valued sequence $x[n] = e^{j2\pi n/2007}$.
- **(A)** Sequence $x[n]$ is periodic with the fundamental period $N_0 = 2\pi$.
- **(B)** Real part of $x[4321]$ is negative.
- **(C)** Imaginary part of $x[-4321]$ is negative.
- **(D)** None of above is true

1.4 The impulse response of LTI system is $h[n] = (-2)^n \mu[-2-n]$
- **(A)** Filter is causal
- **(B)** Filter is stable
- **(C)** Filter is FIR
- **(D)** None of above is true

1.5 The linear convolution $z[n] = x[n] \circledast y[n]$ of sequences $x[n] = (-1)^{n-1}\mu[n-1]$ and $y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$:
- **(A)** $z[n]$ is 3 units long
- **(B)** First non-zero value of $z[n]$ is at $n = -1$
- **(C)** $z[2007] = -1/2007$
- **(D)** $z[2007] = 0$

1.6 Let us know the input $x[n] = 2\delta[n] + 4\delta[n-1] + \delta[n-2] + 5\delta[n-3]$ and the output $y[n] = 2\delta[n] + 4\delta[n-1] + 3\delta[n-2] + 9\delta[n-3] + \delta[n-4] + 5\delta[n-5]$ of an LTI system.
- **(A)** Filter $h[n]$ is bandstop
- **(B)** The length of the impulse response $h[n]$ is the sum of lengths of $x[n]$ and $y[n]$ minus 1
- **(C)** The maximum of the amplitude response $|H(e^{j\omega})|$ is 1.
- **(D)** The impulse response is $h[n] = \delta[n] + 2\delta[n-1] + 4\delta[n-2]$

1.7 Let us compute the linear convolution $y[n] = h[n] \circledast x[n]$. of the impulse response $h[n] = \delta[n-1] + 4\delta[n-2] - 2\delta[n-4] + \delta[n-5]$ and the input $x[n] = \delta[n-1] - 0.84\delta[n-2] - 0.231\delta[n-3] + 0.72\delta[n-5] - \delta[n-6] + \delta[n-7]$.
- **(A)** $y[n] = \{\underline{0},\ 1,\ 3.16,\ -3.591,\ -2.924,\ 3.4,\ 1.502,\ -3.231,\ 2.56,\ 2.72,\ -2,\ 1\}$
- **(B)** $y[n] = \{\underline{0},\ 0,\ 1,\ 3.16,\ -5.591,\ 2.476,\ 1.502,\ -4.671,\ 6.72,\ -2,\ 1\}$
- **(C)** $y[n] = \{\underline{0},\ 0,\ 1,\ 3.16,\ -3.591,\ -2.924,\ 3.4,\ 1.502,\ -3.231,\ 2.56,\ 2.72,\ -3,\ 1\}$
- **(D)** $y[n] = \{\underline{0},\ 0,\ 1,\ 3.16,\ -3.591,\ -2.924,\ 3.4,\ 1.502,\ -3.231,\ 2.56,\ 2.72,\ -2,\ 1\}$

1.8 The highest frequency component in a continuous-time signal is $f_h = 10$ kHz. In order to avoid aliasing
- **(A)** the signal is filtered with a suitable "anti-aliasing" filter
- **(B)** the sampling period is more than $T_s = 0.1$ ms
- **(C)** the sampling period is more than $T_s = 0.05$ ms
- **(D)** we are using the largest sampling frequency that a normal PC soundcard provides

1.9 In Figure 1 there is the spectrum $|X(j\Omega)|$ of analog real-valued signal. When it is sampled with $f_s = 8192$ Hz,
- **(A)** there is no aliasing in the sampling process
- **(B)** 5 kHz component folds down to frequency 3192 Hz
- **(C)** 9 kHz component folds down to frequency 808 Hz
- **(D)** it is possible to reconstruct the original frequency components form the sequence $x[n]$

1.10 Examine the filter $H(z) = (1 - z^{-8})/(1 + 0.8z^{-8})$.
- **(A)** The amplitude response is in Figure 2(a), where x-axis is ω/π
- **(B)** The pole-zero plot is in Figure 2(b)
- **(C)** Filter is 8th order FIR
- **(D)** Impulse response $h[n]$ is nine units long

1.11 z-transform of the output is

$$Y(z) = \frac{1 + z^{-1} + z^{-2}}{1 + 0.81z^{-2}}$$

when an input with z-transform $X(z) = 1 + z^{-1} + z^{-2}$ is fed into a LTI system
- **(A)** Filter is lowpass
- **(B)** The amplification is zero at frequencies $\omega = 0$ and $\omega = \pi$
- **(C)** Impulse response is $h[n] = (0.9j)^n \mu[n]$, when n is even or zero, and $h[n] = 0$, when n is odd
- **(D)** Impulse response is $h[n] = (-0.81)^{n-2}\mu[n-2]$

1.12 Let us know DFT-components $X_1[k] = \{\underline{5},\ 1,\ 1,\ 1\}$ and $X_2[k] = \{\underline{-1},\ 1,\ 3,\ 1\}$ of two sequences, and the corresponding inverse discrete Fourier transforms (IDFT) $x_1[n] = \{\underline{2},\ 1,\ 1,\ 1\}$ and $x_2[n] = \{\underline{1},\ -1,\ 0,\ -1\}$. Compute $X_3[k] = X_1[k] + X_2[k]$ and then IDFT $x_3[n]$:

(A) $x_3[n] = \{\underline{3},\ 0,\ 1,\ 0\}$
(B) $x_3[n] = \{\underline{2},\ 0,\ 0,\ 7\}$
(C) $x_3[n] = \{\underline{1},\ 2,\ 3,\ 4,\ 5\}$
(D) None of above is true

1.13 Linear and time-invariant 2nd order FIR filter has double zero at 0.5, i.e., the zeros are $z_{1,2} = 0.5$.

(A) Filter is two-point moving average
(B) Filter coefficients are complex-valued
(C) The impulse response is infinitely long
(D) None of above is true

1.14 Let us generate a sequence $x[n]$ using Matlab and the sampling frequency 8000 Hz. As the sequence is played from loudspeakers, which of the following gives loudest sine-like "beep" sound? The sequence is a sinusoidal $x[n] = A \cdot \cos(\omega_0 n + \theta)$ with certain amplitude A, normalized angular frequency ω_0 and phase shift θ. The function `ones` returns a vector of ones, e.g. `ones(1,3)` gives a row vector of three ones.

```
n = [0 : 8000];
% here either (A), (B), (C), or (D)
soundsc(x, 8000);   % play from loudspeakers fs = 8000 Hz
```

(A) `x = 0.8 * ones(1, length(n)); % -> [1000 1000 ...  1000 1000]`
(B) `x = 0.1 * cos(0.4 * pi * n + pi/4)`
(C) `x = -0.2 * cos(2 * pi * n)`
(D) `x = 0.5 * cos(0.000125 * pi * n)`

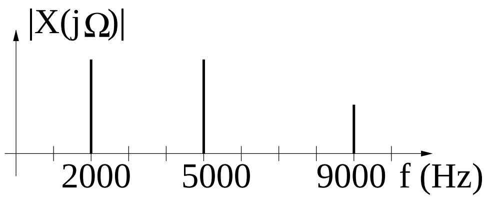

Figure 1: Figure of multichoice Problem 1.9.

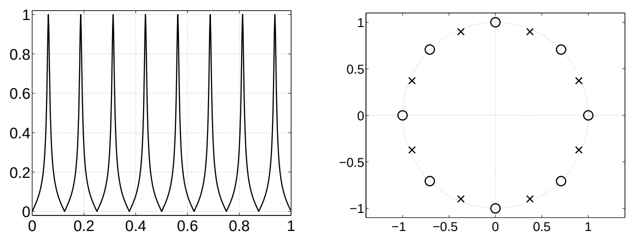

Figure 2: Figures of multichoice Problem 1.10.

2) Let us examine the filter in Figure 3, which consists of two LTI systems H_1 and H_2 and a scaling factor K.

a) Consider a stable and causal H_1

$$H_1(z) = \frac{1}{1 - 0.8z^{-1}}$$

Draw the pole-zero plot of $H_1(z)$, and sketch the amplitude response in range $\omega \in (-\pi \ldots + \pi]$, and write down the impulse response $h_1[n]$.

b) Spectrum can be shifted in frequency domain, when we utilize the formula $H(e^{j(\omega-\omega_0)}) \leftrightarrow e^{j\omega_0 n} h[n]$. Shift H_1 by π:

$$H_2(e^{j\omega}) = H_1(e^{j(\omega-\pi)})$$

when, for instance, zero frequency of H_1 is at π in H_2. Write down expressions for both frequency response $H_2(e^{j\omega})$ and impulse response $h_2[n]$. Compute the first non-zero values of the impulse response $h_2[0]$, $h_2[1]$, and $h_2[2]$.

c) Connect H_1 and H_2 as shown in Figure 3. Write down $H(z)$ in format

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

when the coefficient K is chosen so that the maximum amplification of the filter is 1, and the coefficients b_i and $a_i \in \mathbb{R}$. Compute, what $\sum_{k=-\infty}^{\infty} h[2k+1]$ is. What kind of filter $H(z)$ is: lowpass / highpass / bandpass / bandstop?

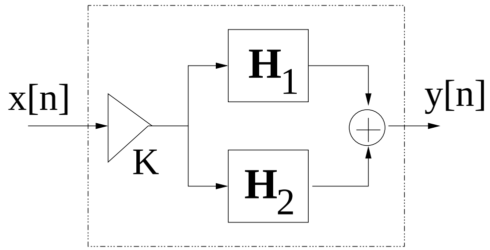

Figure 3: Problem 2: Block diagram of LTI-system $H(z)$, which consist of LTI subsystems $H_1(z)$ and $H_2(z)$ and scaling factor K.