Tik-61.246 Digital Signal Processing and Filtering (DISKO)

2nd Mid Term Exam, November 15, 2000 at 17-20.
You may use a (graphical) calculator and a mathematics handbook.

1. (6p) Are the following statements right or wrong? A right answer gives +1 point, no answer 0 points, and a wrong answer -0.5 points. However, the point minimum for this problem is still zero.

   a) An allpass filter is always linear phase.

   b) The order of the filter $H(z) = \frac{1-2z^{-1}+z^{-2}}{1+0.5z^{-1}}$ is three.

   c) Suppose f_s is the sampling frequency. All frequencies in the range $f_s \dots 3f_s/2$ are aliased to the range $0 \dots f_s/2$.

   d) Aliasing does not occur if the highest frequency component in a signal is less than twice the sampling frequency.

   e) The FFT (Fast Fourier Transform) algorithm approximates DFT (Discrete Fourier Transform) and gives equal results only with an infinite sampling frequency.

   f) Truncating a discrete signal (input sequence) e.g. using the windowing technique causes distortion in the spectrum of the signal.

2. (6p) The one-sided spectrum $X(j\omega)$ of a periodic continuous-time signal $x(t)$ is represented in Figure 1. Let us assume that all components of the signal have the same phase (zero).

   a) Write down the signal $x(t)$ in time-domain as a sum of three cosines (inverse Fourier series)
   $x(t) = \sum_{k=1}^{3} A_k cos(2\pi f_k t + 0)$

   b) What is the length of the basic period $T_0 = 1/f_0$ of the signal?

   c) The continuous-time signal is sampled with a sampling frequency $f_s = 10$ kHz. Sketch the spectrum of the resulting discrete signal.

   d) The continuous-time signal in Figure 1 is first filtered with the following anti-aliasing filter:

$$H(j\omega) = \begin{cases} 1 & |f| < 5 \text{ kHz} \\ 0 & |f| > 6 \text{ kHz} \end{cases}$$

   After that, the resulting signal is sampled with $f_s = 10$ kHz. Sketch the spectrum of the resulting discrete signal.

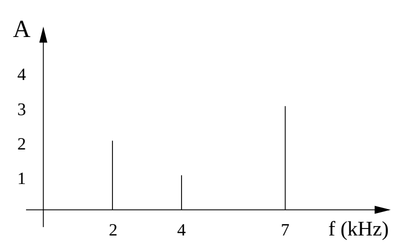

Figure 1: The spectrum in Problem 2.

3. (6p) Let us examine the allpass-type filter illustrated in Figure 2.

   a) Derive the difference equation of the filter.
   b) Derive the transfer function $H(z)$ of the filter.
   c) Sketch a canonic version of the filter (i.e. a version in which the number of delay blocks equals the order of the filter).

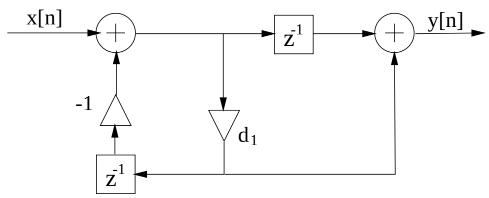

Figure 2: The block structure in Problem 3.

4. (6p) Quantization error can be compensated using error feedback. In the error feedback method, the filtered error signal is added to the branch preceding quantization ($Q[.]$) in the filter structure. Without the feedback, the error signal **would** be the actual quantization error (i.e. $e[n] = y[n] - x[n]$). In the compensated structure, the error signal equals the difference between the output $y[n]$ and the compensated input $w[n]$. In Figure 3, a first-order error feedback structure is represented.

   a) Derive the transfer function $H_e(z)$ of the noise
$$E_{tot}(z) = H_e(z)E(z)$$
   where $E(z)$ is the z-transform of the error $e[n](= Q[w[n]] - w[n])$ and $E_{tot}(z)$ the z-transform of $e_{tot}(z)(= y[n] - x[n])$.
   b) Determine the amplitude response of the transfer function $H_e(z)$ when $b = 1$. Sketch the amplitude response. How does the spectrum of the noise change if the original noise is assumed to be equally distributed to all frequencies?
   c) What happens to the variance of the total error with this structure? (Hint: $\delta_{tot}^2 = \delta_e^2 \sum_{n=-\infty}^{\infty} |h[n]|^2$)

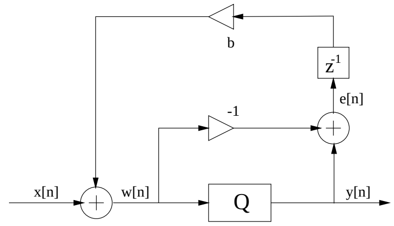

Figure 3: Error feedback structure of Problem 4.

Some z-transforms:
$ax[n - n_0] \leftrightarrow ae^{-j\omega n_0}X(e^{j\omega})$
$H(z) = 1/(1 - e^{-T}z^{-1}) \leftrightarrow h[n] = e^{-nT}\mu[n]$