T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Mon 10.5.2009 9-12. Hall M.

You can do MTE2 only once either 10.5. or 19.5. Mid term exam: Problems 1 and 2.

You can do final exam only once either 10.5. or 19.5. Final exam: Problems 3, 4, 5, 6, and 7. Begin each problem from a new page.

You are not allowed to have any calculator nor math formula book of your own. You will be given a course formula paper. Problem 1 is filled in a specific form.

1) (10 x 1p, 0-9 p, **ONLY MTE2**) Multichoice. There are 1–4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Causal and stable LTI filter

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.3z^{-1} + 0.4z^{-2}}$$

is depicted in a canonic (with respect to delays) direct form II in

(A) Figure 1(a).
(B) Figure 1(b).
(C) Figure 1(c).
(D) Figure 1(d).

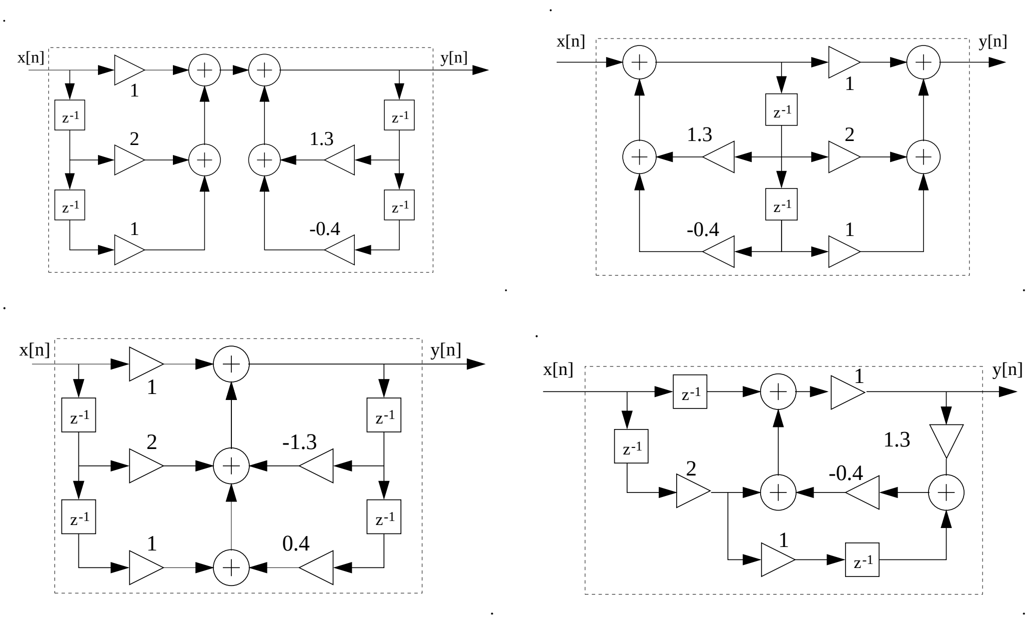

Figure 1: Multichoice 1.1: structures, top row **(A)** and **(B)** , bottom row **(C)** and **(D)** .

1.2 Digital LTI system, with input $x[n]$ and output $y[n]$, and a temporary variable $w[n]$, is expressed with difference equation pairs

$$\begin{aligned} w[n] &= 0.6x[n] + 0.8w[n-1] \\ y[n] &= 0.3x[n-1] - 0.5w[n-1] \end{aligned}$$

(A) There is no feedback, so the system is FIR
(B) There is a feedback without delay ("delay free loop"), and therefore it cannot be realized
(C) The impulse response is $h[n] = (-0.24) \cdot (0.8)^{n-2}\mu[n-2]$
(D) After simplifying with z-transform you get $Y(z) = -0.4z^{-2}X(z)$

1.3 Transfer function of a (monotonic) highpass filter is $H(z) = K \cdot (1 - 2z^{-1} + z^{-2})$. If the maximum of the filter is scaled to one, then the coefficient K has to be

(A) $K = 0.25$
(B) $K = 0.5$
(C) $K = 4$
(D) $K = \infty$

1.4 Filter specifications are given in Figure 2. The sampling frequency is $fT = 12000$ Hz. You may say about specifications:

(A) -3 dB cut-off frequency is at $f_c = 3500$ Hz

(B) Maximum attenuation in stopband is 25 dB

(C) Specifications would tighten and the order of the designed filter would increase, if the stopband cut-off were $f_s = 3500$ Hz

(D) These are specifications for a bandpass filter

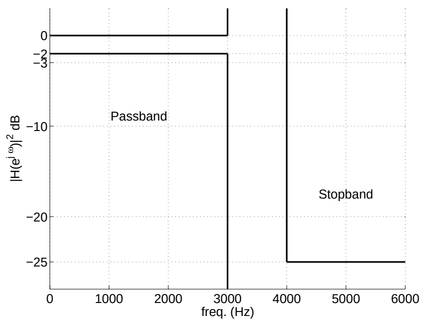

Figure 2: Statement 1.4 specifications.

1.5 Bilinear transform

(A) is bijection (one-to-one mapping), where right plane of the analog s-plane is mapped to inside area of unit circle in z-plane

(B) is a transform, with which quantization noise $E_{tot}(z)$ is mapped to stopband of a filter

(C) one way to compute analog FIR filter from corresponding digital filter

(D) is a mapping, where a digital filter $H(z)$ is received a corresponding analog $H(s)$ so that stable $H(s)$ is always stable $H(z)$

1.6 Stable analog filter $H(s) = \Omega/(s + \Omega)$, with prewarped $\Omega = k \cdot 0.25$, is changed to digital $H(z)$ using $s = k \cdot (1 - z^{-1})/(1 + z^{-1})$. The digital filter will be

(A) $H(z) = 1/(1 + 4k^{-1}z^{-1})$

(B) $H(z) = 0.2 \cdot (1 + z^{-1})/(1 - 0.6z^{-1})$

(C) $H(z) = 0.25 \cdot (1 - z^{-1})/(1.25 - 0.75z^{-1})$

(D) $H(z) = (1 + z^{-1})/(1 + 0.25z^{-1})$

1.7 Impulse response of an ideal highpass filter ($H_{\mathrm{HP}}(z) = 1 - H_{\mathrm{LP}}(z)$) is

$$\begin{aligned} h_{\mathrm{d,HP}}[n] &= \delta[n] - h_{\mathrm{d,LP}}[n] \\ &\approx \{\ldots, 0.0121, -0.1391, \underline{0.3}, -0.1391, 0.0121, \ldots\} \\ h_{\mathrm{d,LP}}[n] &= (\omega_c/\pi) \cdot \mathrm{sinc}(\omega_c n/\pi) \end{aligned}$$

and Hamming window is $w_{\mathrm{Hamming}}[n] = \{0.08, 0.54, \underline{1}, 0.54, 0.08\}$. Construct a filter with "window method" and delay it so that it will be causal.

(A) Transfer function of a highpass filter is $H(z) = 0.08 + 0.54z^{-1} + z^{-2} + 0.54z^{-3} + 0.08z^{-4}$

(B) Phase response of a highpass filter is not linear

(C) Cut-off frequency of a highpass filter is $\omega_c = 0.7\pi$

(D) Magnitude response of a highpass filter $|H(e^{j\omega})|^2$ is in Figure 3.

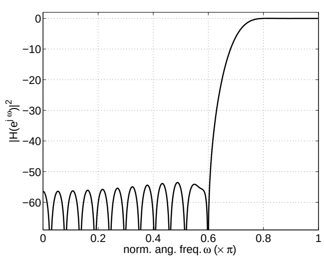

Figure 3: Statement 1.7: **(D)** $|H(e^{j\omega})|^2$

1.8 Fast Fourier Transform (FFT)

(A) requires some N^2 complex operations with large values of N

(B) computes discrete Fourier transform (DFT) always more accurate if there are more computing power available

(C) computes discrete Fourier transform (DFT) with less operations than with definition of DFT

(D) was developed in 1970s in Japanese Tukey Electronics Inc. in order to improve computation in the video card of a video console

1.9 The sampling frequency of a digital sequence is to be increased to (5/3) of the original. When having proper decimation filter $H_D(z)$ ("anti-alias") and interpolation filter $H_I(z)$ ("anti-imaging"), what is the correct way to do this?

(A) $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 3} \rightarrow \boxed{\uparrow 5} \rightarrow y[n]$

(B) $x[n] \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{\downarrow 3} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$

(C) $x[n] \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{H_I(z) \cdot H_D(z)} \rightarrow \boxed{\downarrow 3} \rightarrow y[n]$

(D) $x[n] \rightarrow \boxed{\downarrow 3} \rightarrow \boxed{H_D(z) \cdot H_I(z)} \rightarrow \boxed{\uparrow 5} \rightarrow y[n]$

1.10 You can enter a row in Matlab prompt, where a sum S is computed from seven numbers

`S = 1 - 0.3 - 0.1 - 0.1 - 0.2 - 0.2 - 0.1`

and which returns `-2.7756e-17` $\approx -2.8 \cdot 10^{-17}$. In addition, a different sum

`S = 1 - 0.3 - 0.1 - 0.2 - 0.1 - 0.1 - 0.2`

gives value `-8.3267e-17`.

(A) By default Matlab uses 16 bit precision in number representation (fixed-point arithmetics), which leads to this "rounding error"

(B) By default Matlab uses floating-point arithmetics, which leads to this "rounding error"

(C) Examples prove that Matlab cannot compute the sum of seven numbers so that the result would be exactly zero

(D) Due to examples, one should send a bug report to Matlab (MathWorks Inc.), and demand that errors are corrected immediately

2) (6p, **ONLY MTE2**) Write an exam essay on either subjects 2A or 2B.

2A) **OPTION A.** FFT algorithms. To give an exact example, you can use "radix-2 DIT FFT" algorithm in lecture slides / Mitra's book, with butterfly equations and and W_N in the formula table. Compute DFT using FFT at least for a sequence ($N = 4$) $x[n] = 5\delta[n] - 2\delta[n-1] - 4\delta[n-2] + \delta[n-3]$.

2B) **OPTION B.** Analysis of finite wordlength effects.

3) (6p, **ONLY FINAL EXAM**) Consider four LTI systems h_1, h_2, h_3 and h_4 which form a system h as shown in Figure 4. The following impulse response are known

$$\begin{aligned} h[n] &= \{\underline{0}, 0, -2, 0, -4, -4, -2, -4\} \\ h_2[n] &= 3\delta[n] - \delta[n-1] + \delta[n-2] \\ h_3[n] &= -\delta[n] + \delta[n-1] + \delta[n-2] \\ h_4[n] &= \delta[n-1] + \delta[n-2] \end{aligned}$$

Determine the missing impulse response $h_1[n]$. Show all required steps.

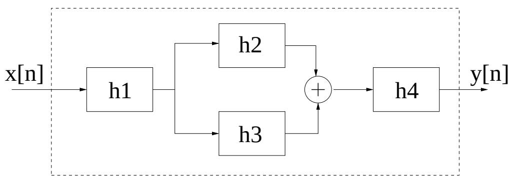

Figure 4: Problem 3. LTI subsystems h_1, h_2, h_3, and h_4 forming a filter h.

4) (6p, **ONLY FINAL EXAM**) Consider a digital LTI filter whose frequency response is

$$H(e^{j\omega}) = \frac{1 + 1.6e^{-j\omega} + 0.68e^{-2j\omega}}{1 - 0.9e^{-j\omega}}$$

a) Explain briefly whether the filter is FIR or IIR.

b) Sketch the pole-zero-plot of the filter.

c) Sketch the magnitude response of the filter. Is it lowpass / highpass / bandpass / bandstop / allpass?

d) Write down the difference equation of the filter.

e) Explain briefly whether the filter is causal or not.

f) Explain or draw one and only one other essential thing about the filter.

5) (6p, **ONLY FINAL EXAM**) Examine a LTI system in Figure 5.

a) Determine the transfer function $H(z)$ as its simpliest form

$$H(z) = K \cdot \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \dots}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots}$$

where constant K is not needed.

b) Draw the block diagram of the filter in canonic (with respect to delays) direct form II structure.

c) Draw the transpose structure of the original filter structure. When transposing structure, "reverse directions", change "branch points to sums" and "sums to branch points".

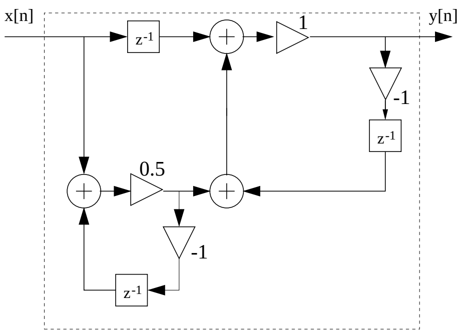

Figure 5: Problem 5. Filter structure.

6) (6p, **ONLY FINAL EXAM**) Analog Butterworth lowpass filter with angular cut-off frequency Ω_c is of form

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$

Compute using bilinear transform a digital first order filter

$$H(z) = K \cdot \frac{1 + f z^{-1}}{1 + g z^{-1}}$$

where constant K is not needed, but write down the numbers f and g. The cut-off of a digital lowpass filter is $f_c =$ 2000 Hz and the sampling frequency $f_T = 10000$ Hz. You can apply the following approximations $\tan(0.1\pi) \approx 0.3$, $\tan(0.2\pi) \approx 0.7$, $\tan(0.3\pi) \approx 1.4$, $\tan(0.4\pi) \approx 3.1$.

7) (6p, **ONLY FINAL EXAM**) Examine stable and causal highpass filter $H(z)$, whose passband starts at 4 kHz and whose sampling frequency is 10 kHz. Magnitude response is given in Figure 6(a) and first values of impulse response $h[n]$ in Figure 6(b).

a) Upsample filter with factor $L = 4$. Sketch both the new amplitude response $H^{'}(z) = H(z^4)$ and impulse response $h^{'}[n] = h[n/4]$ for first 15 values.

b) The filter $H^{'}(z)$ is filtered with a lowpass one, whose cut-off is 10 kHz. Sketch the magnitude response $|H^{''}(e^{j\omega})|$ and depict what kind of filter you have now.

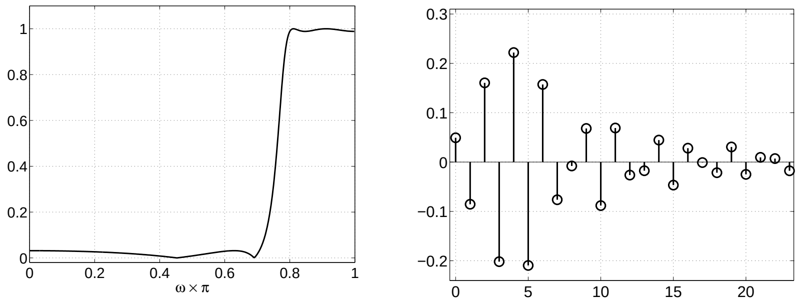

Figure 6: Problem 7. Magnitude response in left, and corresponding impulse response in right.