T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Wed 19.5.2009 15-18. Halls D, E.

You can do MTE2 only once either 10.5. or 19.5. Mid term exam: Problems 1 and 2.

You can do final exam only once either 10.5. or 19.5. Final exam: Problems 3, 4, 5, 6, and 7. Begin each problem from a new page.

You are not allowed to have any calculator nor math formula book of your own. You will be given a course formula paper. Problem 1 is filled in a specific form.

1) (10 x 1p, 0-9 p, **ONLY MTE2**) Multichoice. There are 1–4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Causal and stable LTI filter

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.3z^{-1} + 0.4z^{-2}}$$

is depicted in a canonic (with respect to delays) direct form II in

(A) Figure 1(a).
(B) Figure 1(b).
(C) Figure 1(c).
(D) Figure 1(d).

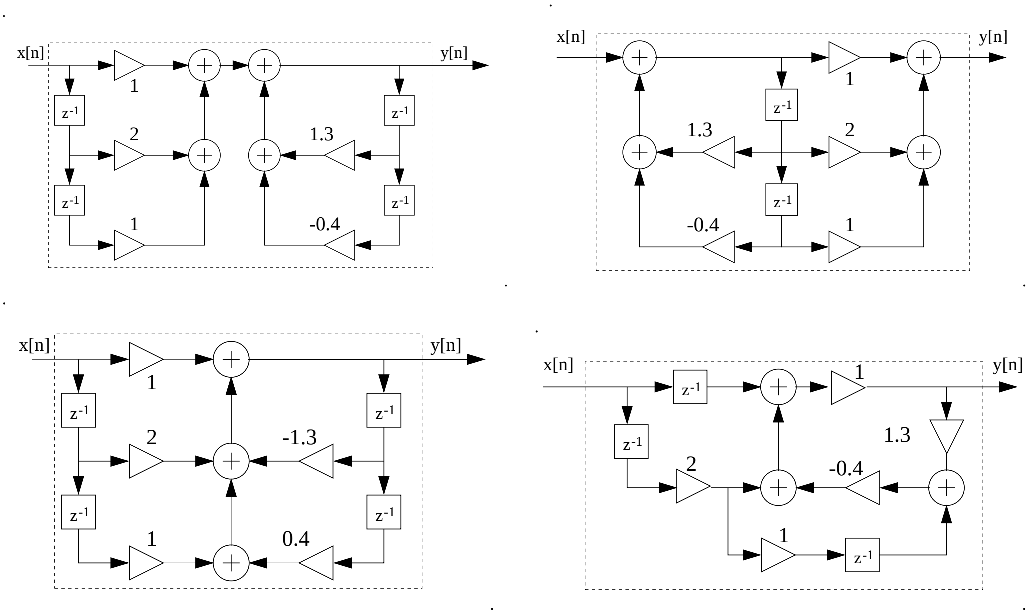

Figure 1: Multichoice 1.1: structures, top row **(A)** and **(B)** , bottom row **(C)** and **(D)** .

1.2 Consider a digital LTI system in Figure 2 with two temporary variables $v[n]$ and $w[n]$ and two coefficients 0.5 and −0.5. Using these a set of difference equations are obtained

$$\begin{aligned} v[n] &= x[n] + w[n] \\ w[n] &= -0.5w[n-1] + y[n-1] \\ y[n] &= 0.5v[n] \end{aligned}$$

(A) Because the filter has been drawn as a direct form I structure, it can be easily seen that the order of the filter is two
(B) Restructuring it to a direct form, it can be seen to be first order FIR filter
(C) Structure is canonical with respect to delays
(D) Impulse response of the filter is $h[n] = \{\underline{0.5}, 0.25, 0.125, 0.0625, \ldots\}$

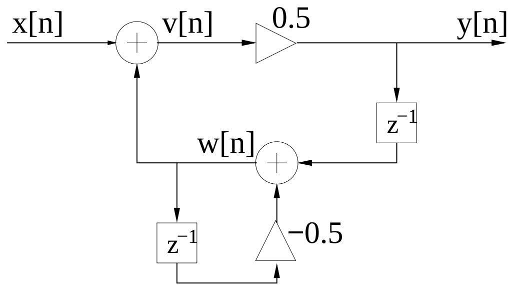

Figure 2: Multichoice 1.2 filter structure.

1.3 The squared magnitude response $|H(e^{j\omega})|^2$ of a LTI system $H(z)$ is given in Figure 3. Its maximum is 20 dB, which can be got

$$10 \cdot \log_{10}(\frac{H_{max}^2}{H_1^2}) = 20 \cdot \log_{10}(\frac{H_{max}}{H_1}) = 20 \text{ dB}$$

where $H_1 = 1$. What is value K so that maximum amplification is $\max\{K \cdot H(z)\} = 1$ or 0 dB?

(A) $K = 1/\infty$

(B) $K = 0.1$

(C) $K = 5/\pi$

(D) $K = 10$

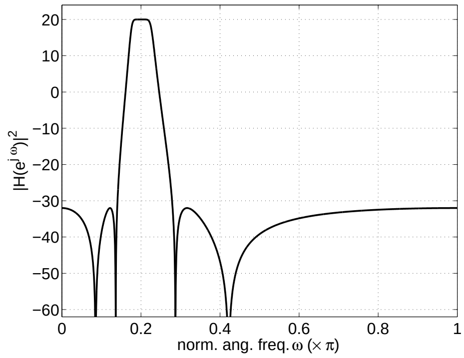

Figure 3: Multichoice 1.3 $|H(e^{j\omega})|^2$

1.4 One basic principle in filter design is

(A) to analyze $H(z)$ from different viewpoints so that one can ensure if the filter is stable or not

(B) to compute coefficients of $H(z)$ so that the magnitude response just meets the given spefications

(C) to estimate first the order of filter using computers (e.g. Matlab), add a small number to the order (+2 tai +4) so that one ensures that coefficients are not quantizised, and after that the final coefficients $H(z) = B(z)/A(z)$ are computed computer-aided

(D) to place a required set of zeros d_m and poles p_n into a pole-zero plot, and compute coefficients of $H(z)$: computer-aided from

$$H(z) = G \cdot \frac{\prod_{m=1}^{M}(1 - d_m z^{-1})}{\prod_{n=1}^{N}(1 - p_n z^{-1})}$$

1.5 Bilinear transform is a one-to-one mapping (bijection) between s-plane (analog filter) and z-plane (digital filter). Let us use sampling period $T = 2$, and transform and inverse are

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}, \qquad z = \frac{1 + s}{1 - s}$$

Consider s-plane point $s = \sigma_0 + j\Omega_0$, hence,

$$\begin{aligned} z &= \frac{1 + (\sigma_0 + j\Omega_0)}{1 - (\sigma_0 + j\Omega_0)} = \frac{(1 + \sigma_0) + j\Omega_0}{(1 - \sigma_0) + j\Omega_0} \\ \Rightarrow |z|^2 &= \frac{(1 + \sigma_0)^2 + (\Omega_0)^2}{(1 - \sigma_0)^2 + (\Omega_0)^2} \end{aligned}$$

(A) If all poles s_i of s-plane are in left plane, the all corresponding poles z_i of a digital filter are also in left plane of z-plane

(B) If real part of a pole $\sigma_0 = 0$ in s-plane, then the pole maps onto z-plane into the same point $z = 1$ regardless of Ω_0

(C) If s-plane pole with real part $\sigma_0 < 0$, then the pole maps onto z-plane inside the circle of radius 1

(D) If all poles s_i of s-plane are in right plane, then all corresponding poles z_i of a digital filter are inside the unit circle in z-plane

1.6 Using command `[B, A] = cheby2(5, 30, 0.25);` you will get Chebychev II -type digital filter, whose order is $N = 5$, stopband minimum attenuation 30 dB and stopband cut-off $\omega_{\text{stop}} = 0.25\pi$. The magnitude response of the filter is in

(A) Figure 4(a)
(B) Figure 4(b)
(C) Figure 4(c)
(D) Figure 4(d)

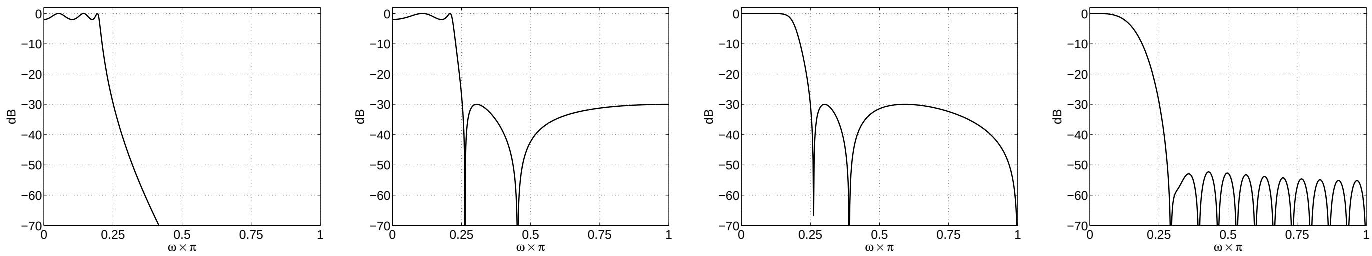

Figure 4: Multichoice 1.6 magnitude responses **(A)** , **(B)** , **(C)** ja **(D)** .

1.7 Impulse response of an ideal highpass filter ($H_{\text{HP}}(z) = 1 - H_{\text{LP}}(z)$) is

$$\begin{aligned} h_{\text{d,HP}}[n] &= \delta[n] - h_{\text{d,LP}}[n] \\ &\approx \{\ldots, 0.0121, -0.1391, \underline{0.3}, -0.1391, 0.0121, \ldots\} \\ h_{\text{d,LP}}[n] &= (\omega_c/\pi) \cdot \text{sinc}(\omega_c n/\pi) \end{aligned}$$

and Hamming window is $w_{\text{Hamming}}[n] = \{0.08, 0.54, \underline{1}, 0.54, 0.08\}$. Construct a filter with "window method" and delay it so that it will be causal.

(A) Transfer function of a highpass filter is $H(z) = 0.08 + 0.54z^{-1} + z^{-2} + 0.54z^{-3} + 0.08z^{-4}$
(B) Phase response of a highpass filter is linear
(C) Cut-off frequency of a highpass filter is $\omega_c = 0.3\pi$
(D) Magnitude response of a highpass filter $|H(e^{j\omega})|^2$ is in Figure 5.

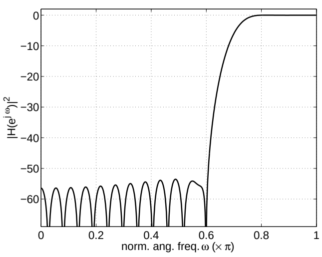

Figure 5: Multichoice 1.7: **(D)** $|H(e^{j\omega})|^2$

1.8 Window functions can be used in cropping a sequence into a certain finite length. In Figure 6 there are rectangular and Blackman windows of length $N = 33$ ($M = 16$, $-M \leq n \leq M$) in both time and frequency domain. After cropping DFT is computed in order to get a spectrum.

(A) Wide main lobe of Blackman window makes detection of close frequency components worse than with rectangular window (narrower main lobe)

(B) Blackman window attenuates first and last values of cropped $x[n]$ so that weak (in amplitude) frequency components do not show in the spectrum as good as with rectangular window

(C) Greater attenuation with side lobes of Blackman window affect that weak (in amplitude) frequency components do not show in the spectrum as good as with rectangular window

(D) Blackman window modifies the cropped sequence $x[n]$ so that frequency components change into frequencies twice as high as original ones

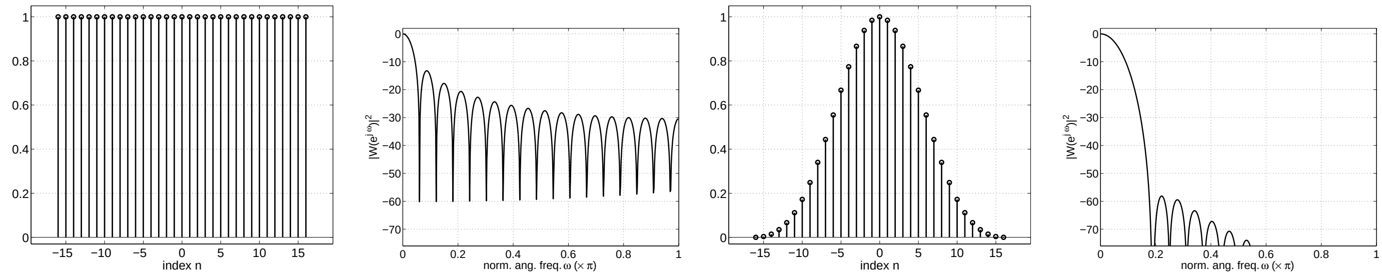

Figure 6: Multichoice 1.8: (a) rectangular window $w_r[n]$ and its (b) magnitude response $|W_r(e^{j\omega})|^2$, (c) Blackman window $w_b[n]$ and its (d) magnitude response $|W_b(e^{j\omega})|^2$.

1.9 Periodic sequence ($N_0 = 6$) $x[n] = \{\ldots, 8, 3, 4, 2, 9, -1, \ldots\}$ is fed into digital system $x[n] \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 2} \rightarrow y[n]$. What comes out?

(A) the same sequence $y[n] = x[n]$

(B) sequence $y[n]$ but with half length

(C) $y[n] = \{\ldots, 8, 0, 4, 0, 9, 0, \ldots\}$

(D) $y[n] = 2x[n]$

1.10 You are running a script function `myFunc1.m` for the first time in Matlab. The first lines of the code:

```
1   [x, fT] = wavread('kiisseli.wav');
2   M       = length(x);
3   N       = 512;
4   overlap = 128;
5   D       = zeros(ceil(M/N), 3);
6   for k = (1 : N-overlap : M-N)
7       ind      = ind+1;
8       D(ind,1) = sum(abs(fft(x(k : k+N-1) .* hamming(N))));
```

Execution stops with an error message in Command window:

```
??? Undefined function or variable 'ind'.

Error in ==> myFunc1 at 7
    ind  = ind+1;
```

What is the best solution to this problem?

(A) Save all files, exit Matlab and start it again. Run `myFunc1.m` again

(B) Feed a command `ind = 0;` in Command window and run the code again

(C) Double-click the error message in Command window and the pointer moves to a line 7 in the editor. You remove the whole line 7, which caused an error

(D) Add a new line `ind = 0;` before row 6 in the editor

2) (6p, **ONLY MTE2**) Write an exam essay on either subjects 2A or 2B.

2A) **OPTION A.** FFT algorithms. To give an exact example, you can use "radix-2 DIT FFT" algorithm in lecture slides / Mitra's book, with butterfly equations and and W_N in the formula table. Compute DFT using FFT at least for a sequence ($N = 4$) $x[n] = 5\delta[n] - 2\delta[n-1] - 4\delta[n-2] + \delta[n-3]$.

2B) **OPTION B.** Analysis of finite wordlength effects.

3) (6p, **ONLY FINAL EXAM**) Consider the cascade of three LTI systems in Figure 7. It is known that

$$\begin{aligned} h_1[n] &= \delta[n-1] - \delta[n-2] \\ h_2[n] &= \delta[n+1] + 2\delta[n] - \delta[n-1] \end{aligned}$$

a) What is the impulse response of the total filter $h[n]$?

b) If the output of the system $h[n]$ is

$$y[n] = -\delta[n+1] + 6\delta[n-1] - 4\delta[n-2] - 7\delta[n-3] + 8\delta[n-4] - 2\delta[n-5]$$

what has been the input $x[n]$?

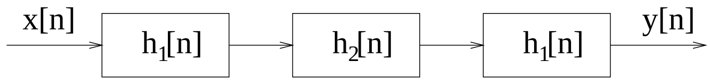

Figure 7: Problem 3 cascade filter.

4) (6p, **ONLY FINAL EXAM**) Consider a digital LTI filter whose transfer function is

$$H(z) = \frac{1 - 1.7z^{-1} + 0.72z^{-2}}{1 - 0.8z^{-1}}$$

a) Explain briefly whether the filter is FIR or IIR.

b) Sketch the pole-zero-plot of the filter.

c) Sketch the magnitude response of the filter. Is it lowpass / highpass / bandpass / bandstop / allpass?

d) Write down the difference equation of the filter.

e) Explain briefly whether the filter is causal or not.

f) Explain or draw one and only one other essential thing about the filter.

5) (6p, **ONLY FINAL EXAM**) Examine spectrum $X(j\Omega)$ of analog signal $x(t)$ in Figure 8.

a) What is the most important thing in Shannon's sampling theorem?

b) If the signal is sampled with $f_T = 9$ kHz, sketch the spectrum $X(e^{j\omega})$ of sampled sequence.

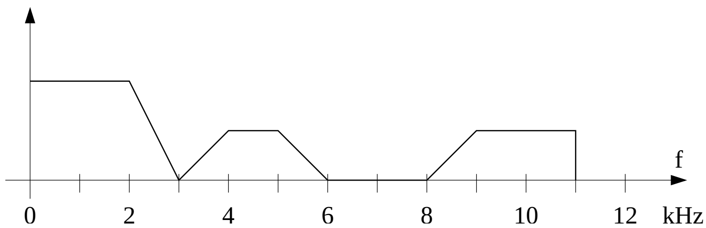

Figure 8: Problem 5. The spectrum $X(j\Omega)$ of analog signal $x(t)$.

6) (6p, **ONLY FINAL EXAM**) Consider a second order LTI system, whose transfer function is

$$H_1(z) = \frac{1 - 1.2z^{-1} + z^{-2}}{1 + 1.5z^{-1} + 0.6z^{-2}}$$

where poles are at $p = -0.75 \pm 0.1936j$ and zeros at $z = 0.6 \pm 0.8j$ and the maximum of the amplitude response is at $\omega = \pi$.

a) Sketch the magnitude response $|H_1(z)|$ in range $[0, \ldots, 2\pi]$

b) Replace delay registers of $H_1(z)$ by double delays $H_2(z) = H_1(z^2)$ (compare to upsampling $L = 2$). Write down the filter $H_2(z)$ in form

$$H_2(z) = K \cdot \frac{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

and sketch the magnitude response $|H_2(z)|$ in range $[0, \ldots, \pi]$.

c) Determine the coefficient K so that the maximum of $H_2(z)$ is scaled to one.

7) (6p, **ONLY FINAL EXAM**) See the filter in Figure 9. The input values are represented with B bits. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B, it is necessary to quantize values of $w[n]$ (block Q).

Quantization error can be compensated using so called error feedback (or error-shaping filter). In Figure 9 there is a second order filter with a second order error feedback system.

Write down first the difference equations for $e[n]$ and $w[n]$, and write down then in frequency domain the quantized output $Y(z)$ using input $X(z)$, filter modifing input $H_x(z)$, quantization noise $E(z)$, and filter modifing noise $H_e(z)$ in form of

$$Y(z) = H_x(z)X(z) + H_e(z)E(z)$$

and reply

a) how does the filter behave, if it is possible to use infinite wordlength, i.e. there is no quantization and $e[n] \equiv 0, \forall n$?

b) how does the spectrum of the total noise $E_{tot}(z) = H_e(z)E(z)$ look like if there is no compensation, i.e. $k = 0$, and if $e[n]$ is white noise so that $E(z) = 1$ for all frequencies?

c) with which simple value of k the effect of noise is suppressed in the passband?

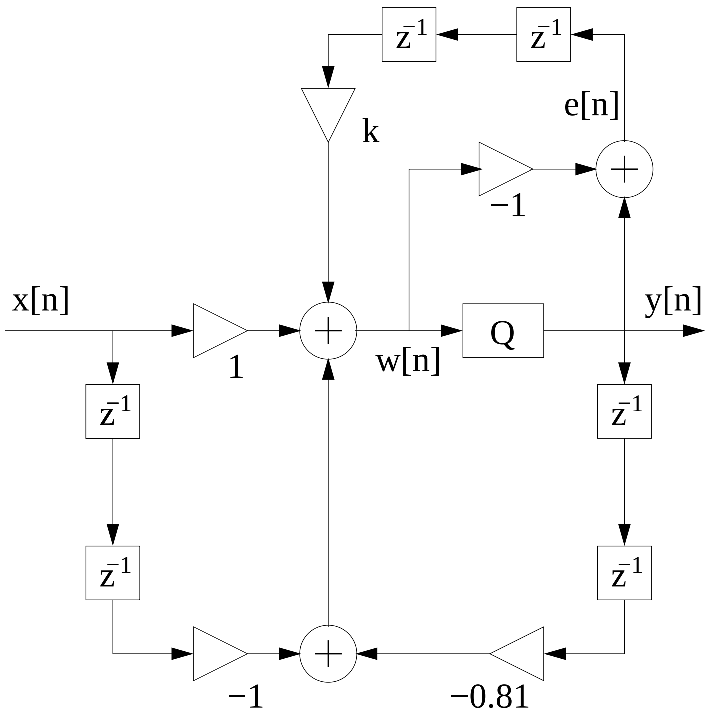

Figure 9: Problem 7. Second order system with second order error feedback.