5 Finite-Length Discrete Transform

Introduction

- In this chapter, ***finite-length transforms*** are discussed
- In practice, it is often convenient to map a finite-length sequence from time domain into a finite-length sequence of the same length in the frequency domain
- The samples of the forward transform are unique and represented as a linear combination of the samples of the time domain sequence
- The samples of the inverse transform are obtained similarly from the samples of the transform domain

Introduction

- In some applications, a very long-length time domain sequence is broken up into a set of short-length sequences and a finite-length transform is applied to each short-length sequence
- The transformed sequences are processed in the transform domain
- Time domain equivalents are produced using the inverse transform
- The processed short-length sequences are grouped together in the time domain to form the final long-length sequence

Orthogonal Transforms

- Let $x[n]$ denote a length-N time domain sequence with $X[k]$ denoting the coefficients of its N-point orthogonal transform
- A general form of the orthogonal transform pair is of the form

$$X[k] = \sum_{n=0}^{N-1} x[n]\psi^*[k,n], \quad 0 \le k \le N-1$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\psi[k,n], \quad 0 \le n \le N-1$$

Orthogonal Transforms

- In the transform pair, the ***basis sequences*** $\psi[k,n]$ are also length-N sequences in both domains
- In the class of finite-dimensional transforms, the basis sequences satisfy the condition

$$\frac{1}{N}\sum_{n=0}^{N-1} \psi[k,n]\ \psi^*[l,n] = \begin{cases} 1, & l = k \\ 0, & l \ne k \end{cases}$$

- Basis sequences $\psi[k,n]$ satisfying the above condition are said to be ***orthogonal*** to each other

Orthogonal Transforms

- An important consequence of the orthogonality of the basis sequence is the energy preservation property of the transform
- The energy $\sum_{n=0}^{N-1}|x[n]|^2$ of the time domain sequence $x[n]$ can be computed in the transform domain
- The energy can be written as

$$\sum_{n=0}^{N-1}|x[n]|^2 = \sum_{n=0}^{N-1} x[n]\ x^*[n]$$

Orthogonal Transforms

- Let us express $x[n]$ in terms of its transform domain representation

$$\sum_{n=0}^{N-1} x[n]\; x^*[n] = \sum_{n=0}^{N-1}\left(\frac{1}{N}\sum_{k=0}^{N-1} X[k]\psi[k,n]\right)\; x^*[n]$$

$$\Rightarrow = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\left(\sum_{n=0}^{N-1} x^*[n]\psi[k,n]\right) = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\; X^*[k]$$

$$\Rightarrow \boxed{\sum_{n=0}^{N-1} \left|x[n]\right|^2 = \frac{1}{N}\sum_{k=0}^{N-1}\left|X[k]\right|^2}$$

which is known as the ***Parseval's relation***

The Discrete Fourier Transform

- In the following, the discrete Fourier transform, DFT, is defined
- The inverse transformation, IDFT is developed
- Some important properties of the DFT are discussed
- DFT has several important applications:
  - Numerical calculation of the Fourier transform in an efficient way
  - Implementation of linear convolution using finite-length sequences

Definition of the DFT

- ***Discrete Fourier transform*** (DFT) of the length-N sequence $x[n]$ is defined by

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad 0 \le k \le N-1$$

- The basis sequences are: $\psi[k,n] = e^{-j2\pi kn/N}$ which are complex exponential sequences
- As a result, DFT coefficients $X[k]$ are complex numbers, even if $x[n]$ are real
- It can be easily shown that the basis sequences $e^{j2\pi kn/N}$ are orthogonal

Discrete Fourier Transform (DFT)

- Common notation with DFT: $W_N = e^{-j2\pi/N}$
- DFT can now be written as follows:

$$X[k] = \sum_{n=0}^{N-1} x[n]\; W_N^{kn}, \quad k = 0,1,\ldots,N-1$$

- ***Inverse Discrete Fourier Transform*** (IDFT):

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\; W_N^{-kn}, \quad n = 0,1,\ldots,N-1$$

- $X[k]$ and $x[n]$ are both sequences of finite-length N

Discrete Fourier Transform Pair

- The analysis equation:

$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n]\; W_N^{kn}, \quad k = 0,1,\ldots,N-1}$$

- The synthesis equation:

$$\boxed{x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\; W_N^{-kn}, \quad n = 0,1,\ldots,N-1}$$

- DFT pair is denoted as: $x[n] \overset{DFT}{\longleftrightarrow} X[k]$

Relation Between the Discrete-Time Fourier Transform and the DFT

- The DTFT $X(e^{j\omega})$ of the length-N sequence $x[n]$ defined for $0 \le n \le N-1$ is given by:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\; e^{-j\omega n} = \sum_{n=0}^{N-1} x[n]\; e^{-j\omega n}$$

- By uniformly sampling $X(e^{j\omega})$ at N equally spaced frequencies $\omega_k = 2\pi k/N$, $0 \le k \le N-1$, on the ω-axis between $0 \le k \le 2\pi$

$$X(e^{j\omega})\Big|_{\omega=2\pi k/N} = \sum_{n=-\infty}^{\infty} x[n]\; e^{-j2\pi nk/N}, \quad 0 \le k \le N-1$$

Relation Between the Discrete-Time Fourier Transform and the DFT

- The N-point DFT sequence $X[k]$ is precisely the set of frequency samples of the Fourier transform $X(e^{j\omega})$ of the length-N sequence $x[n]$ at N equally spaced frequencies, $\omega_k = 2\pi k/N$, $0 \le k \le N-1$
- Hence, the DFT $X[k]$ represents a frequency domain representation of the sequence $x[n]$
- Since the computation of the DFT samples involve a finite sum, for time domain sequences with finite sample values, the DFT always exists

Numerical Computation of the Fourier Transform Using the DFT

- The DFT provides a practical approach to the numerical computation of the Fourier transform of a finite-length sequence
- Let $X(e^{j\omega})$ be the Fourier transform of a length-N sequence $x[n]$
- We wish to evaluate $X(e^{j\omega})$ at a dense grid of frequencies $\omega_k = 2\pi k/M$, $0 \le k \le M-1$, where $M >> N$:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\ e^{-j\omega_k n} = \sum_{n=0}^{N-1} x[n]\ e^{-j2\pi kn/M}$$

Numerical Computation of the Fourier Transform Using the DFT

- Define a new sequence $x_e[n]$ obtained from $x[n]$ by augmenting with M-N zero-valued samples

$$x_e[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ 0, & N \le n \le M-1 \end{cases}$$

- Making use of $x_e[n]$ we obtain

$$X(e^{j\omega_k}) = \sum_{n=0}^{M-1} x_e[n]\ e^{-j2\pi kn/M}$$

which is an M-point DFT $X_e[k]$ of the length-M sequence $x_e[n]$

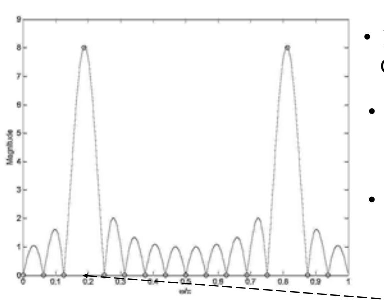
Example 5.5:

- Compute the N-point DFT of the length-16 sequence $x[n]=\cos(6\pi n/16)$ of angular frequency ω_0=0.375π
- The 16-point DFT of $x[n]$ is (Example 5.2)

$$X[k] = \begin{cases} 8, & \text{for } k = 3 \text{ and } k = 13 \\ 0, & \text{otherwise} \end{cases}$$

- Since the Fourier transform $X(e^{j\omega})$ is a continuous function of ω, we can plot it more accurately by computing the DFT of the sequence $x[n]$ at a dense grid of frequencies using MATLAB

Example 5.5: $x[n]=\cos(6\pi n/16)$



- 16-point DFT denoted by 'o'
- 512-point DFT denoted by '___'
- Normalized frequency with $2\pi = 1$;

3/16 = 0.1875

Sampling the Fourier Transform

- The discrete Fourier transform DFT can also be obtained by sampling the discrete-time Fourier transform DTFT, $X(e^{j\omega})$, uniformly on the ω-axis between $0 \le \omega \le 2\pi$, at $\omega_k = 2\pi k/N$, $k=0,1,\ldots,N-1$

$$X[k] = X(e^{j\omega})\Big|_{\omega=2\pi k/N} = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0,1,\ldots,N-1$$

- $X[k]$ is now a finite-length sequence of length N like the time domain sequence $x[n]$

Operations on Finite-Length Sequences

- Like the Fourier transform, the DFT also satisfies a number of properties that are useful in signal processing
- Some of the properties are essentially identical to those of the Fourier transform, while some others are different
- Differences between two important properties are discussed:
  - Shifting and
  - Convolution

Circular Shift of a Sequence

- Several DFT properties and theorems involve shifting in the time domain and in the frequency domain
- The operation of shifting of a finite-length sequence in time domain is referred to as ***circular time-shifting***
- In frequency domain the corresponding operation is referred to as ***circular frequency-shifting***

Circular Shift of a Sequence

- Consider length-N sequences defined for the range $0 \le n \le N-1$
- Such sequences have zero for $n<0$ and $n \ge N$
- Shifting such a sequence $x[n]$ for any arbitrary integer n_0, the resulting sequence $x_1[n]=x[n-n_0]$ is no longer defined for the range $0 \le n \le N-1$
- It is necessary to define a shift operation that will keep the shifted sequence in the range $0 \le n \le N-1$
- This is achieved using the ***modulo operation***

Circular Shift of a Sequence

- Let $0, 1, ..., N$ be a set of N positive integers and let m be any integer
- The integer r obtained by evaluating m modulo N is called the ***residue*** and it is an integer with a value between 0 and $N-1$
- The modulo operation is denoted as

$$\langle m \rangle_N = m \text{ modulo } N$$

- If we let $r = \langle m \rangle_N$, then $r = m + lN$ where l is an integer to make $m+lN$ a number in the range $0 \le n \le N-1$

Circular Shift of a Sequence

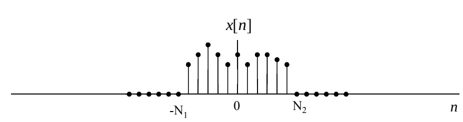
- Using the modulo operation, the circular shift of a length-N sequence $x[n]$ can be defined using the equation

$$x_C[n] = x[\langle n - n_0 \rangle_N]$$

where $x[n]$ is also length-N sequence
- The concept of circular shift of a finite-length sequence corresponds to "rotation" of the sequence within the interval $0 \le n \le N-1$

Representation of a Finite-Length Sequence

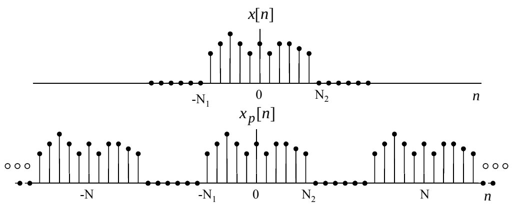
- Consider a general sequence $x[n]$ that is of ***finite-length***, i.e., for some integers N_1 and N_2, $x[n] = 0$ outside the range $-N_1 \le n \le N_2$



- The shifting operation of finite-length sequences can be represented via periodic sequences

Representation of Aperiodic Signals

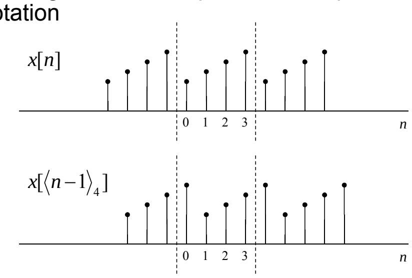
- A periodic sequence, $x_p[n]$, is formed from the ***aperiodic*** sequence with $x[n]$ as one period



- As N approaches infinity, $x_p[n] = x[n]$ for any finite value n

Circular Time-Shift of a Sequence

- Shifting of a finite sequence corresponds to rotation



Circular Convolution

- Consider two length-N sequences, $g[n]$ and $h[n]$
- Their linear convolution is a sequence of length $2N$-1

$$y_L[n] = \sum_{m=0}^{N-1} g[m]h[n-m], \quad n = 0,1,...,2N-1$$

- In order to calculate the above linear convolution both length-N sequences have been zero-padded to extend their length to $2N$-1

Circular Convolution

- A convolution-like operation resulting in a length-N sequence $y_C[n]$, called a ***circular convolution*** is defined as

$$y_C[n] = \sum_{m=0}^{N-1} g[m]h[\langle n-m\rangle_N]$$

- The above operation is often referred to as an ***N-point circular convolution***
- Due to length-N sequences, the N–point circular convolution is denoted as

$$y_C[n] = g[n] \textcircled{N} h[n] = h[n] \textcircled{N} g[n]$$

Application of Circular Convolution

- The ***N-point circular convolution*** does not correspond to the ***linear convolution*** of two length-N sequences
- The circular convolution can, however, be used to compute the linear convolution correctly:
  - The linear convolution of two finite-length sequences of length N and M results in a sequence of length $N+M$-1
  - The circular convolution must be computed for the length $N+M$-1 by zero-padding the original sequences

Classification of Finite-Length Sequences

- For a finite-length sequence defined for $0 \le n \le N$-1, all definitions of symmetry do not apply
- The definitions of symmetry in the case of finite-length sequences are given such that the symmetric and antisymmetric parts of length-N sequence are also of length N and defined for the same range of values of the time index n

Classification Based on Geometric Symmetry

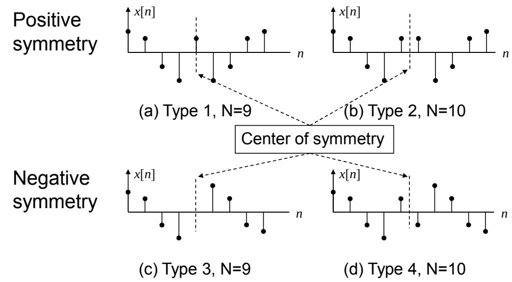
- Geometric symmetry is an important property in DSP, i.e., in the properties of FIR filters
- A length-N ***symmetric sequence*** $x[n]$ satisfies the condition

$$x[n] = x[N\text{-}1\text{-}n]$$

- A length-N ***antisymmetric sequence*** $x[n]$ satisfies the condition

$$x[n] = -x[N\text{-}1\text{-}n]$$

Geometric Symmetry of Sequences



Type 1 Symmetry with Odd Length

- Type 1 symmetric sequence, with N=9, is

$$x[n] = x[0] + x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8]$$

- The Fourier transform is

$$X\left(e^{j\omega}\right) = x[0] + x[1]e^{-j\omega} + x[2]e^{-j2\omega} + x[3]e^{-j3\omega} + x[4]e^{-j4\omega} + x[5]e^{-j5\omega} + x[6]e^{-j6\omega} + x[7]e^{-j7\omega} + x[8]e^{-j8\omega}$$

- Now, $x[0]=x[8]$, $x[1]=x[7]$, $x[2]=x[6]$, $x[3]=x[5]$

$$X\left(e^{j\omega}\right) = x[0]\left(1 + e^{-j8\omega}\right) + x[1]\left(e^{-j\omega} + e^{-j7\omega}\right) + x[2]\left(e^{-j2\omega} + e^{-j6\omega}\right) + x[3]\left(e^{-j3\omega} + e^{-j5\omega}\right) + x[4]e^{-j4\omega}$$

Type 1: Symmetry with Odd Length

- Taking $e^{-j4\omega}$ as a common factor in each group of terms

$$X\left(e^{j\omega}\right) = x[0]e^{-j4\omega}\left(e^{j4\omega} + e^{-j4\omega}\right) + x[1]e^{-j4\omega}\left(e^{j3\omega} + e^{-j3\omega}\right) + x[2]e^{-j4\omega}\left(e^{j2\omega} + e^{-j2\omega}\right) + x[3]e^{-j4\omega}\left(e^{j\omega} + e^{-j\omega}\right) + x[4]e^{-j4\omega}$$

$$X\left(e^{j\omega}\right) = e^{-j4\omega}\left\{x[0]\left(e^{j4\omega} + e^{-j4\omega}\right) + x[1]\left(e^{j3\omega} + e^{-j3\omega}\right) + x[2]\left(e^{j2\omega} + e^{-j2\omega}\right) + x[3]\left(e^{j\omega} + e^{-j\omega}\right) + x[4]\right\}$$

$$\Rightarrow \quad X\left(e^{j\omega}\right) = e^{-j4\omega}\left\{2x[0]\cos(4\omega) + 2x[1]\cos(3\omega) + 2x[2]\cos(2\omega) + 2x[3]\cos(\omega) + x[4]\right\}$$

Type 1: Symmetry with Odd Length

- Notice that the quantity inside the braces, { }, is a real function of ω and can assume positive or negative values in the range $0 \le \omega \le \pi$
- The of the sequence is given by $\theta(\omega) = -4\omega + \beta$ where β is either 0 or π, and hence the phase is a linear function of ω
- In general, for Type 1 linear-phase sequence of length-N

$$X\left(e^{j\omega}\right) = e^{-j(N-1)\omega/2}\left\{x\left[\tfrac{N-1}{2}\right] + 2\sum_{n=1}^{(N-1)/2} x\left[\tfrac{N-1}{2} - n\right]\cos(\omega n)\right\}$$

Type 2: Symmetry with Even Length

- Similarly, the Fourier transform of Type 2 symmetric sequence, with N=8, can be written

$$X\left(e^{j\omega}\right) = e^{-j7\omega/2}\left\{2x[0]\cos(\tfrac{7\omega}{2}) + 2x[1]\cos(\tfrac{5\omega}{2}) + 2x[2]\cos(\tfrac{3\omega}{2}) + 2x[3]\cos(\tfrac{\omega}{2})\right\}$$

where the phase is given by $\theta(\omega) = -\frac{7\omega}{2} + \beta$

- In general, for Type 2 linear-phase sequence of length-N

$$X\left(e^{j\omega}\right) = e^{-j(N-1)\omega/2}\left\{2\sum_{n=1}^{N/2} x\left[\tfrac{N}{2} - n\right]\cos\left(\omega\left(n - \tfrac{1}{2}\right)\right)\right\}$$

Type 3: Antisymmetry with Odd Length

- The Fourier transform of Type 3 antisymmetric sequence, with N=9, is (notice that $x[4]$=0)

$$X(e^{j\omega}) = x[0] + x[1]e^{-j\omega} + x[2]e^{-j2\omega} + x[3]e^{-j3\omega} + x[4]e^{-j4\omega} + x[5]e^{-j5\omega} + x[6]e^{-j6\omega} + x[7]e^{-j7\omega} + x[8]e^{-j8\omega}$$

- Now, $x[0]$=-$x[8]$, $x[1]$=-$x[7]$, $x[2]$=-$x[6]$, $x[3]$=-$x[5]$ and $x[4]$=0

$$X(e^{j\omega}) = x[0](1 - e^{-j8\omega}) + x[1](e^{-j\omega} - e^{-j7\omega}) + x[2](e^{-j2\omega} - e^{-j6\omega}) + x[3](e^{-j3\omega} - e^{-j5\omega})$$

$$X(e^{j\omega}) = e^{-j4\omega}\{x[0](e^{j4\omega} - e^{-j4\omega}) + x[1](e^{j3\omega} - e^{-j3\omega}) + x[2](e^{j2\omega} - e^{-j2\omega}) + x[3](e^{j\omega} - e^{-j\omega})\}$$

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Type 3: Antisymmetry with Odd Length

- Multiplying by $j=e^{j\pi/2}$ and 2, we obtain

$$X(e^{j\omega}) = e^{-j4\omega}e^{j\pi/2}\{2x[0]\tfrac{1}{2j}(e^{j4\omega} - e^{-j4\omega}) + 2x[1]\tfrac{1}{2j}(e^{j3\omega} - e^{-j3\omega}) + 2x[2]\tfrac{1}{2j}(e^{j2\omega} - e^{-j2\omega}) + 2x[3]\tfrac{1}{2j}(e^{j\omega} - e^{-j\omega})\}$$

which results in

$$X(e^{j\omega}) = e^{j(-4\omega+\pi/2)}\{2x[0]\sin(4\omega) + 2x[1]\sin(3\omega) + 2x[2]\sin(2\omega) + 2x[3]\sin(\omega)\}$$

The phase is now $\theta(\omega) = -4\omega + \frac{\pi}{2} + \beta$

- The antisymmetry introduces a phase shift of $\pi/2$

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Type 3 and 4: Antisymmetry with Odd and Even Length

- In general, the Fourier transform of Type 3 linear-phase antisymmetric sequence of odd length-N is

$$X(e^{j\omega}) = je^{-j(N-1)\omega/2}\left\{2\sum_{n=1}^{(N-1)/2} x[\tfrac{N-1}{2} - n]\sin(\omega n)\right\}$$

- Similarly, the Fourier transform of Type 4 linear-phase antisymmetric sequence of even length-N is

$$X(e^{j\omega}) = je^{-j(N-1)\omega/2}\left\{2\sum_{n=1}^{N/2} x[\tfrac{N}{2} - n]\sin(\omega(n - \tfrac{1}{2}))\right\}$$

- In both cases, $j=e^{j\pi/2}$ introduces a phase shift of $\pi/2$

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Discrete Fourier Transform Theorems

- The important theorems hold for DFT with time domain sequences length-N and their DFTs of length-N , e.g.,
  - Linearity
  - Circular time-shifting
  - Circular frequency-shifting
  - Circular convolution
  - Modulation
  - Parseval's theorem
- The proofs are straightforward using the definitions

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Linear Convolution of Two Finite-Length Sequences

- Let $g[n]$ and $h[n]$ be two finite-length sequences of lengths N and M, respectively
- The objective is to implement their linear convolution

$$y_L[n] = g[n] \circledast h[n]$$

- The length of the sequence $y_L[n]$ is $L=N+M-1$
- The linear convolution can be obtained using the circular convolution with the correct length equal to L

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Linear Convolution of Two Finite-Length Sequences

- Define two length-L sequences $g_e[n]$ and $h_e[n]$ by appending $g[n]$ and $h[n]$ with zero-valued samples

$$g_e[n] = \begin{cases} g[n], & 0 \le n \le N-1 \\ 0, & N \le n \le L-1 \end{cases}$$

$$h_e[n] = \begin{cases} h[n], & 0 \le n \le M-1 \\ 0, & M \le n \le L-1 \end{cases}$$

- Then,

$$y_L[n] = y_C[n] = g_e[n] \, \textcircled{L} \, h_e[n]$$

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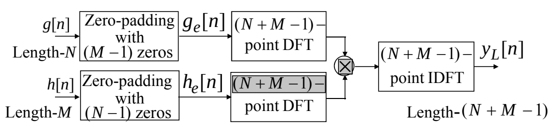
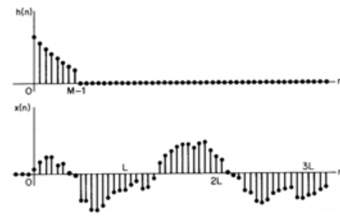
Linear Convolution of Two Finite-Length Sequences Using the DFT

- The linear convolution of two finite-length sequences $g[n]$ and $h[n]$ can be implemented using the DFTs of length $L=N+M-1$ as follows



Data Sequence of Unknown Length

- Problem: Filtering of a data sequence of unknown, or infinite length with an FIR filter, with impulse response, $h[n]$, of length M using the DFT



Linear Convolution of Finite-Length Sequences

- Filtering of a data sequence of unknown (infinite) length with an FIR filter, with impulse response, $h[n]$, of length M can be implemented via circular convolution, i.e., using the DFT
- The data sequence $x[n]$ is first segmented into finite-length sections of length-L
- Two methods to implement the linear convolution
  - Overlap-add method
  - Overlap-save method

Overlap-Add Method

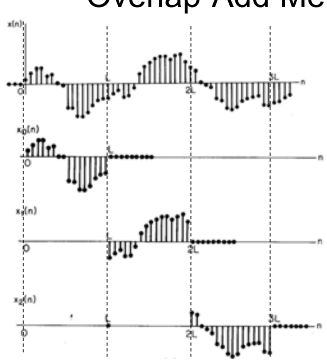
- The causal data sequence $x[n]$ is first segmented into segments of length L
- The original sequence $x[n]$ can now be written as

$$x[n]=\sum_{m=0}^{\infty} x_m[n-mL]$$

where

$$x_m[n]=\begin{cases} x[n+mL], & 0\le n\le L-1 \\ 0, & \text{otherwise} \end{cases}$$

Overlap-Add Method



- Original sequence, $x[n]$, of unknown length
- Non-overlapping length-L segments of $x[n]$
- Adding the segments gives

$$x[n]=\sum_{m=0}^{\infty} x_m[n-mL]$$

Overlap-Add Method

- Substituting the segmented form of $x[n]$ into the convolution sum

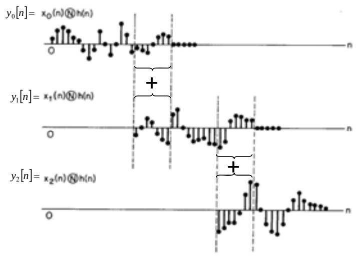
$$y[n]=\sum_{k=0}^{M-1} h[k]x[n-k]=\sum_{k=0}^{M-1} h[k]\sum_{m=0}^{\infty} x_m[n-k-mL]$$

$$=\sum_{m=0}^{\infty}\left(\sum_{k=0}^{M-1} h[k]x_m[n-k-mL]\right)=\sum_{m=0}^{\infty} y_m[n-mL]$$

where $y_m[n]=h[n] \circledast x_m[n]$

- The linear convolutions of $h[n]$ and the segments of $x_m[n]$, which all are all of length-N, ($N=M+L-1$) are thus added

Overlap-Add Method



- The linear length-N convolutions of $h[n]$ and $x_m[n]$
- The overlapping parts of the linear convolutions are added

$$y[n] = \sum_{m=0}^{\infty} y_m[n - mL]$$

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Overlap-Save Method

- It is possible to implement the linear convolution also by performing circular convolutions of length shorter than (M+L-1)
- In this case, it is necessary to segment the original sequence $x[n]$ is into overlapping blocks $x_m[n]$,
- The terms of the circular convolution of $h[n]$ with $x_m[n]$ that correspond to the terms obtained by a linear convolution of $h[n]$ and $x_m[n]$
- The other, incorrect, terms of the circular convolution are thrown away

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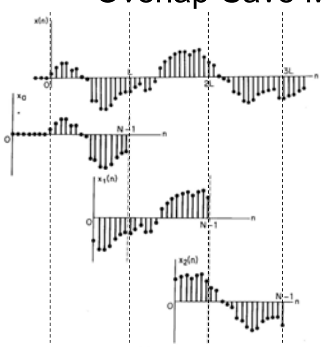
Overlap-Save Method



- Original sequence, $x[n]$, of unknown length
- Overlapping length-N segments of $x_m[n]$
- Circular convolution is implemented with length N

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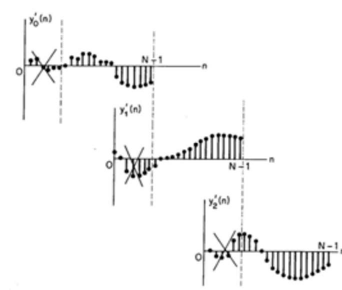
Overlap-Save Method



- The length-N circular convolutions of length-M impulse response, $h[n]$, and the blocks $x_m[n]$ of length-N
- The incorrect M-1 first terms in each circular convolution are rejected

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Summary

- The discrete Fourier transform, DFT, of a finite-length sequence was discussed
- The length of the transform coefficient sequence, i.e., the length of the DFT, is the same as the length of the discrete-time sequence
- The DFT is widely used in a number of digital signal processing applications
- In practice, the DFT can be efficiently implemented using the Fast Fourier Transform (FFT) algorithm

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