T-61.140 Signal Processing Systems

1st mid term exam, Monday 8th March 2004, 15-18, hall M.
You are NOT ALLOWED to use any mathematical reference book or calculator. Some tables given on additional paper. **Show intermediate steps in Problems 2-4.**

1) (6p) Statements, answer either TRUE (T) or FALSE (F). Correct answer +1 point, a wrong −1 point. Reply to as many statements as you want. However, the maximum points of the problem is six, and the minimum 0 points. No explanations needed.

a:		b:		c:		d:		e:		f:		g:		h:		i:	

a) A sequence $x[n] = e^{3\pi jn/5 - j\pi/4} + e^{-3\pi jn/5 + j\pi/4}$ can be represented with a sine function: $x[n] = 2\sin(3\pi n/5 + \pi/4)$.

b) A sequence $x[n] = \sum_{k=-\infty}^{+\infty}(\delta[n-3k] - \delta[n+2k-1])$ is periodic and its fundamental period is $N_0 = 6$.

c) The fundamental period T_0 of the signal $x(t) = \cos(10\pi t) + \sin(20\pi t)$ is 5 Hz.

d) The output $y[n]$ of a causal LTI system is zero for all $n < 0$.

e) Consider a LTI system, whose impulse response is $h[n] = 0.5^n u[1-n]$.
Statement: Filter is stable.

f) $y(t) = 2x(t-\pi)$ is a causal LTI system.

g) It was shown in Matlab exercises that a median filter can efficiently remove random white and black spots from a gray-scale picture. This means that high-frequency noise is reduced. M-point median filter sorts M gray-scale values ($x[n]$, $x[n-1]$, …, $x[n-M+1]$) in ascending order and picks up the middle one.
Statement: The median filter is a LTI system of lowpass type.

h) The convolution of sequences $x[n] = 2004(\delta[n-2004] + \delta[n-2005])$ and $h[n] = \delta[n] + \delta[n+1]$ is
$y[n] = h[n] * x[n] = 2004\delta[n-2004] + 4008\delta[n-2005] + 2004\delta[n-2006]$.

i) The inverse transform of the spectrum $Y(j\omega) = e^{-2j\omega}/(1+0.5j\omega)$ is
$y(t) = 2e^{-2(t-2)}u(t-2)$.

2) (6p) Draw a flow (block) diagram and write down the difference equation of the recursive, feedback IIR-system, whose impulse response is of infinite length

$$h[n] = 4\delta[n] - \delta[n-1] + 0.5\delta[n-2] - 0.25\delta[n-3] + 0.125\delta[n-4] - 0.0625\delta[n-5] + \dots$$

There are two or three multipliers (triangle), one or two delays (memory registers, a square with D), and one or two sum elements (circle with +) in the diagram depending on the realization.

3) (6p) Consider a linear, time-invariant, stable and causal discrete-time system, where the input $x[n]$ and output $y[n]$ are:

n	$x[n]$	$y[n]$
0	1	2
1	-2	1
2	0	?
3	1	?
4	2	?
5	0	?

a) (4p) Define the impulse response $h[n]$ of the system using $x[n]$ and $y[n]$, and conditions that initial values of system are zero and it is form (a, b, c and d constants):

$$h[n] = \begin{cases} a, & \text{when } n < 0 \\ b, & \text{when } n = 0 \\ c, & \text{when } n = 1 \\ d, & \text{when } n > 1 \end{cases}$$

b) (2p) Calculate the missing values of $y[n]$.

4) (6p) A periodic signal $x(t)$ with fundamental period $T_0 = 100$ seconds is defined:

$$x(t) = \begin{cases} 0, & 0 \leq t < 40 \\ 2, & 40 \leq t < 50 \\ 4, & 50 \leq t < 60 \\ 2, & 60 \leq t < 100 \end{cases}$$

a) (1p) Draw the signal in range $-20 < t < 120$ seconds.

b) (1p) What is the fundamental angular frequency Ω_0 rad/s?

c) (3p) Compute the Fourier-coefficients a_k.

d) (1p) Signal $x(t)$ is approximated by the signal $x_{aM}(t)$, which has only a part (finite number) of Fourier-series terms:

$$x_{aM}(t) = \sum_{k=-M}^{M} a_k \, e^{jk(2\pi/T_0)t}$$

Draw the approximating signal $x_{a0}(t)$, which has only the coefficient a_0, into the same figure as in (a).