T-61.6070 Exercise problem 20.4.

Consider a simplified version of the MMSB method with two categories and the same membership distribution $\theta_i = \theta = \begin{bmatrix} \phi \\ 1-\phi \end{bmatrix}$ for all nodes i.

Assume that the adjacency matrix r and the categories z_{ij} and z_{ji} for all node pairs (i, j) are known. Particularly, the number of zs which equal to 1 is Z_1 and the number of zs which equal to 2 is Z_2.

The generative process is

- $\theta \sim$ Uniform
- $z_{ij} \sim$ Multinomial(θ)
- $z_{ji} \sim$ Multinomial(θ)
- $r_{ij} \sim$ Bernoulli($\eta_{z_{ij} z_{ji}}$)

The likelihood is $p(r, z|\eta, \theta) = \prod_i \prod_{j>i} p(z_{ij}|\theta) p(z_{ji}|\theta) p(r_{ij}|z_{ij}, z_{ji}, \eta)$.

Task: Compute maximum a posteriori (MAP) estimate for ϕ, i.e. the value of ϕ that maximizes the posterior $p(\theta, \eta|r, z) = \frac{p(r,z|\eta,\theta)p(\eta)p(\theta)}{p(r,z)}$.

Hint: When computing the MAP estimate, everything in the posterior that does not depend on θ (or ϕ) can be regarded as constant.