# MODEL STRUCTURE SELECTION

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## Linear Model Structures

A system is linear if it is possible to describe it by a model of the form:

$y(t) = G(q^{-1})u(t) + H(q^{-1})e(t)$

where H and G are linear operator with time delay operator

e(t) is white noise signal independent of past inputs.

The **model structure** *M* is a parameterized set of candidate models

$M \Rightarrow \{G(q^{-1}, \theta), \; H(q^{-1},\theta) \mid \theta \; \varepsilon \; D_m \}$

for system $y(t) = G(q^{-1},\theta) \, u(t) + H(q^{-1},\theta) \, e(t)$

where θ denotes adjustable parameters in n dimension and Dm is subset of parameters of n dimension

The model is a particular choice of parameter vector $\theta = \theta'$

## Different types of Linear Model structures

1. The Finite Impulse Response model structure

2. ARX model structure ( AutoRegressive, external input)

3. ARMAX model structure (Autoregressive, Moving Average, external input)

4. Output Error (OE) model

5. Box-Jenskins (BJ) model

## The Finite Impulse response model structure

The simplest model structure where

$G(q^{-1},\theta) = q^{-d}B(q^{-1})$ $\qquad$ $H(q^{-1},\theta) = 1$

The predictor is given by

$\hat{y}(t|\theta) = q^{-d}B(q^{-1})\, u(t)$ ,

*predicted output* $\qquad$ *input*

This system has no poles and is stable

ARX model structure (Autoregressive, external output)

The matrix G has poles as output is also dependent on past inputs

so $G(q\text{-}1,\theta) = q^{-d} \dfrac{B(q^{-1})}{A(q^{-1})}$ and $H(q\text{-}1,\theta) = \dfrac{1}{A(q^{-1})}$

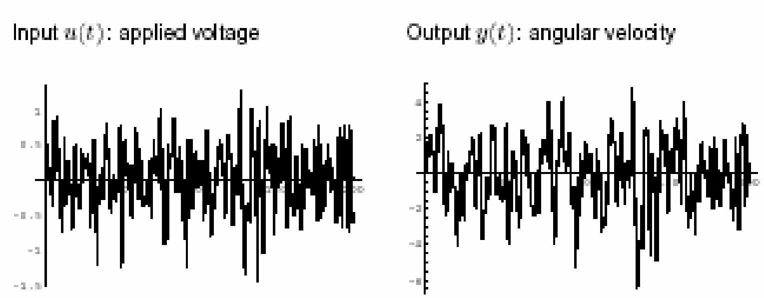
The predictor is always stable and is given by

$\hat{y}(t|\theta) = q^{-d}B(q^{-1})\, u(t) + [1 - A(q^{-1})]\, y(t)$

Example: Identification of a DC motor

The system can be written as

*y(t) = a y(t-1) + b u(t-1)*



## ARMAX model structure (Autoregressive, moving average, external input)

The matrix G and H both have poles and zeros as output is dependent on past and present inputs as well as past prediction error

so $G(q^{-1},\theta) = q^{-d} \dfrac{B(q^{-1})}{A(q^{-1})}$ and $H(q^{-1},\theta) = \dfrac{C(q^{-1})}{A(q^{-1})}$

The optimal predictor is :

$\hat{y}(t|\theta) = q^{-d}B(q^{-1})u(t) + [1 - A(q^{-1})]y(t) + [C(q^{-1}) - 1]\,\varepsilon(t,\theta)$

**represents prediction error as $\varepsilon(t,\theta) = y - \hat{y}(t|\theta)$**

The predictor has poles which needs to be inside unit circle for a stable prediction

example: balancing of airoplanes in flight using movement from desired tilt as error.

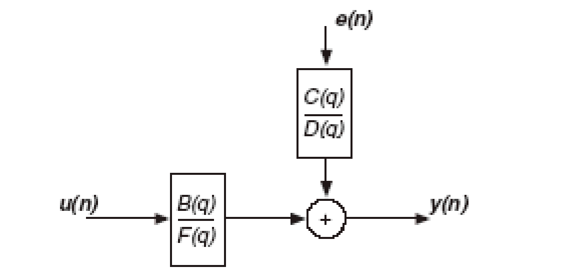
## Output error model structure (OE)

If the noise affecting the system is white measurement noise then matrix H can written as $H(q-1,\theta) = 1$ where as matrix still remains as $G = q^{-d} \dfrac{B(q^{-1})}{F(q^{-1})}$

The predictor is: $\hat{y}(t|\theta) = q^{-d}B(q^{-1})u(t) + [1 - F(q^{-1})]\, y(t)$

The Box-Jenkins (BJ) structure provides a complete model with disturbance properties modeled separately from system dynamics.

Box-Jenkins Model Structure




The Box-Jenkins model is useful when you have disturbances that enter late in the process.
For example, measurement noise on the output is a disturbance late in the process.

## SSIF (State Space Innovations Form)

It is an alternative to input-output model structure

When the state of the system is represented by ***x*(t)**.

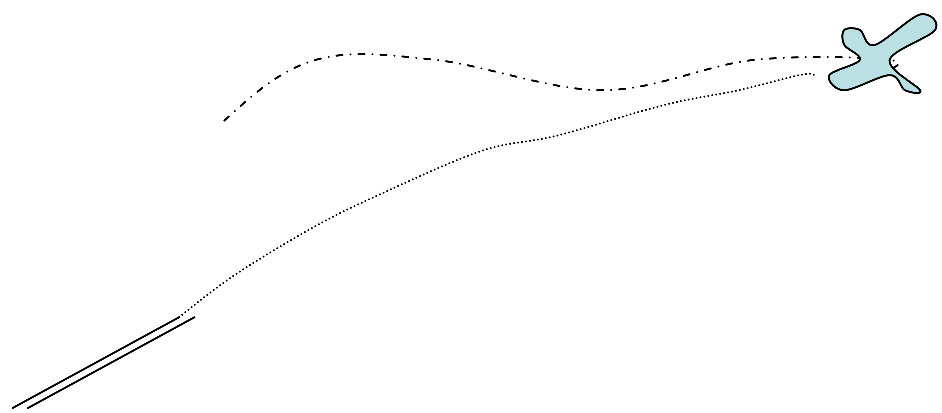
State update and output follow the equations

$$x(t+1) = A(\theta)x(t) + B(\theta)u(t) + w(t)$$

$$y(t) = C(\theta)x(t) + v(t)$$

**independent white noise**

predictor for linear state space model optimal predictor is known as kalman filter which can be summarised



# The Discrete Kalman Filter

UPDATE (prediction) → Correction (filtering)

**UPDATE (prediction)**

(1) Project the state ahead

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k^- = AP_{k-1}A^T + Q$$

(1) Compute the Kalman gain

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

(2) Update estimate with measurement $y_k$

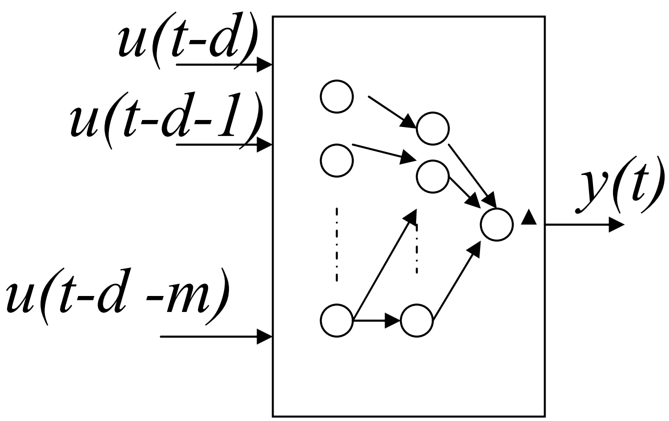
$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - H\hat{x}_k^-)$$

(3) Update the error covariance

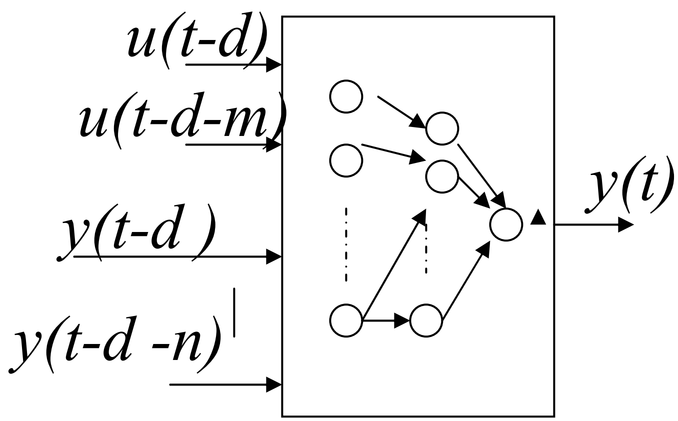
$$P_k = (I - K_k H)P_k^-$$

Where $P_k$ is estimation error covariance and $K_k$ is Kalman gain.

## Nonlinear Model Structures based on neural network



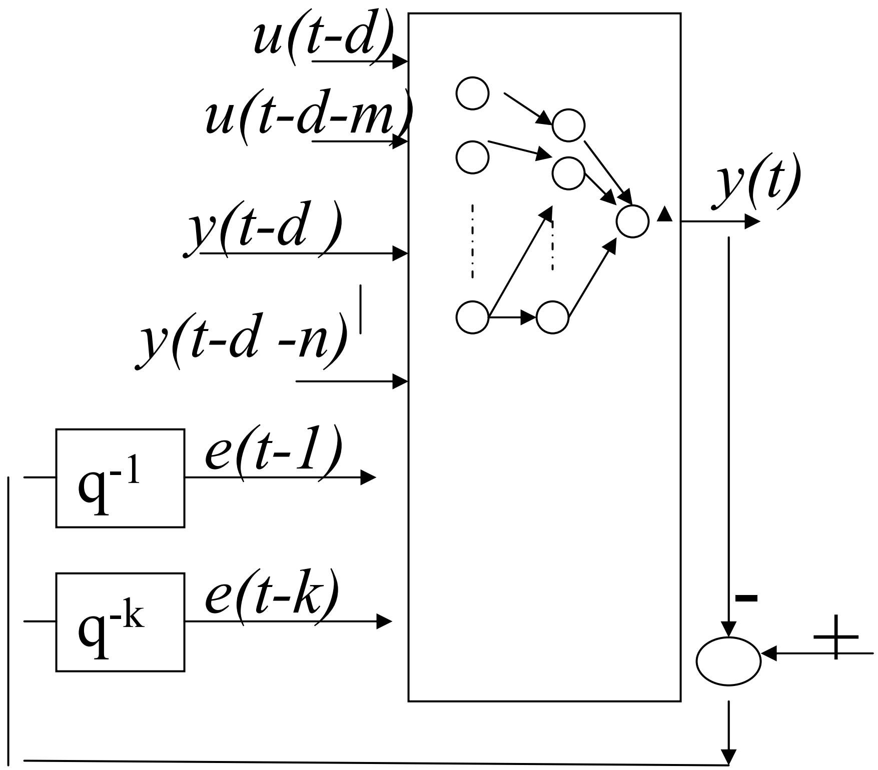

**1. NNFIR**

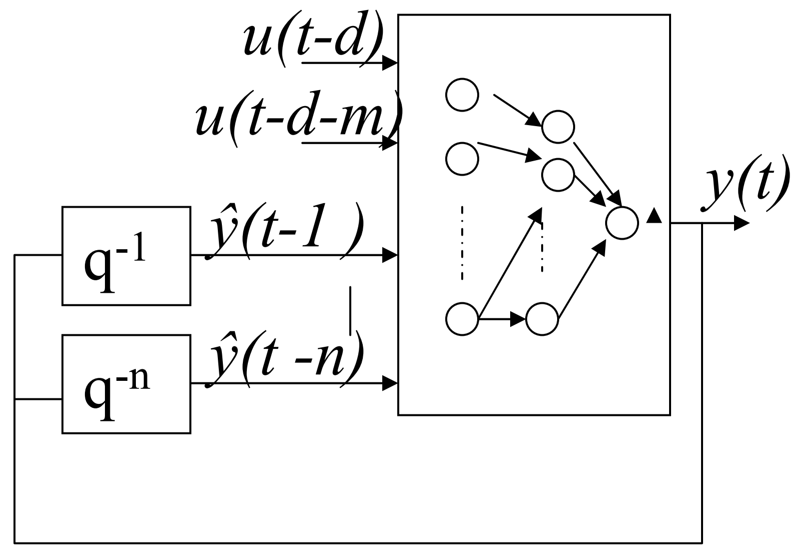



**2. NNARX**

NNFIR and NNARX systems are free of noise so both systems are deterministic

## Nonlinear Model Structures based on neural network



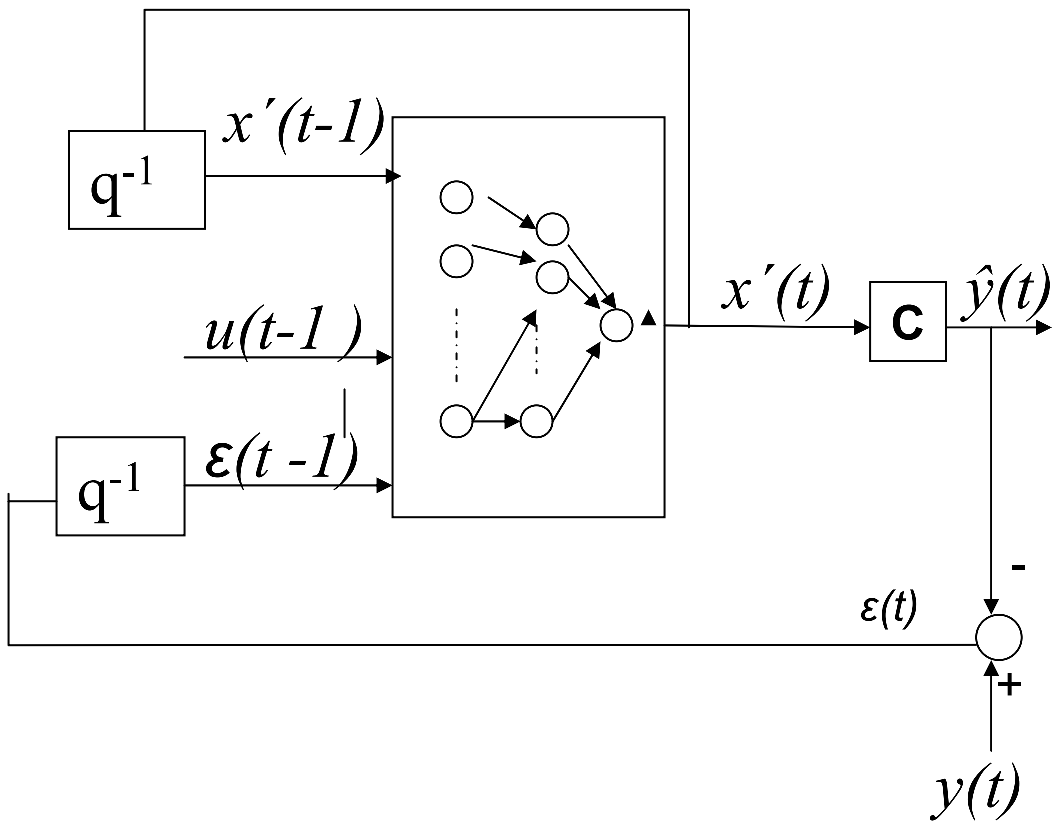




**4. NNOE**

3. **NNARMAX**

They take noise in to account. These model are more difficult to design than that of **NNARX** model, **as** these models are recurrent.

## Nonlinear Model Structures based on neural network




Is not straight forward to apply.

## Hybrid model structure:

In this way, some prior knowledge can be incorporated into the black box model to reduce its complexity and improve its adaptation and prediction properties.

Hybrid models of linear and non-linear systems can be designed. hybrids can be made of

Regressions
Neural Networks
Fuzzy Logic based models