T-61.184
Automatic Speech Recognition: From Theory to Practice

http://www.cis.hut.fi/Opinnot/T-61.184/

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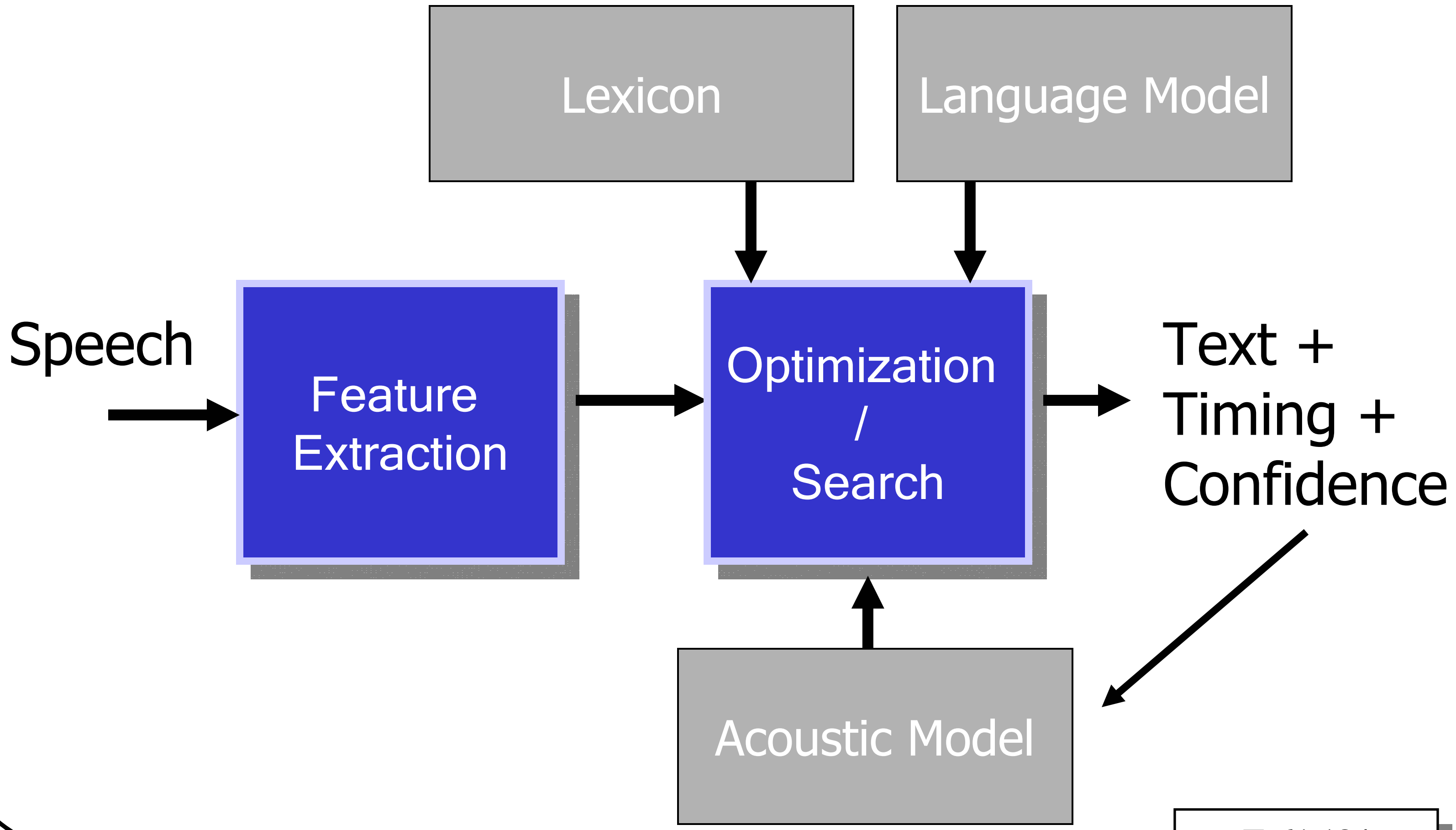
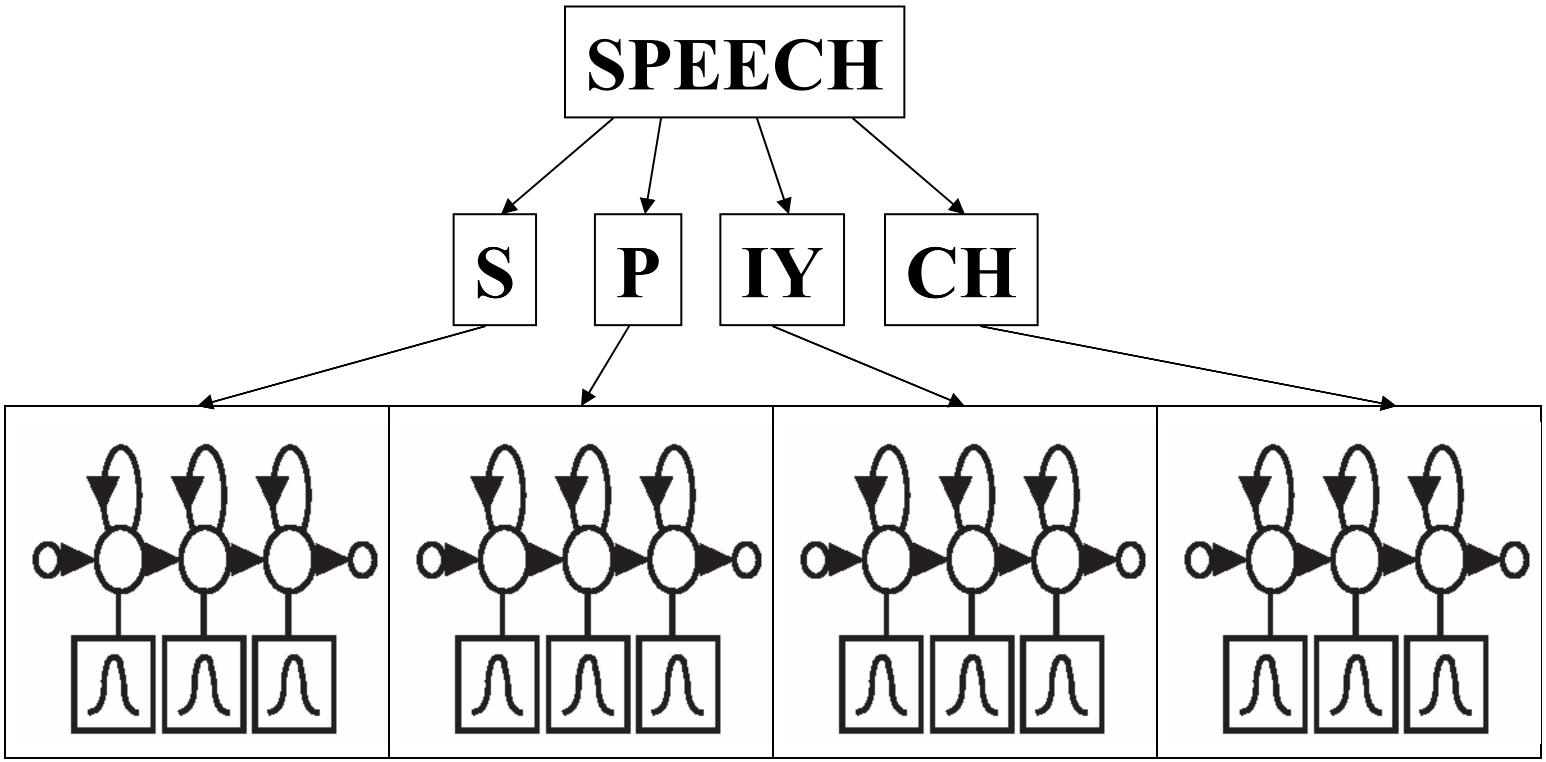
Role of Hidden Markov Models in Speech Recognition



Motivation for Today's Topic



Expectations for this course

- **Today to present a basic idea of Hidden Markov Models (HMMs)**
- **Next week we'll discuss how HMMs are used in practice to model acoustics of speech**
- **Don't be lost in the math today: I don't expect you'll get the theory from all these slides! If you want to learn about HMMs it's important to take a look at the resources listed on the next slide.**

Resources for Today's Topic

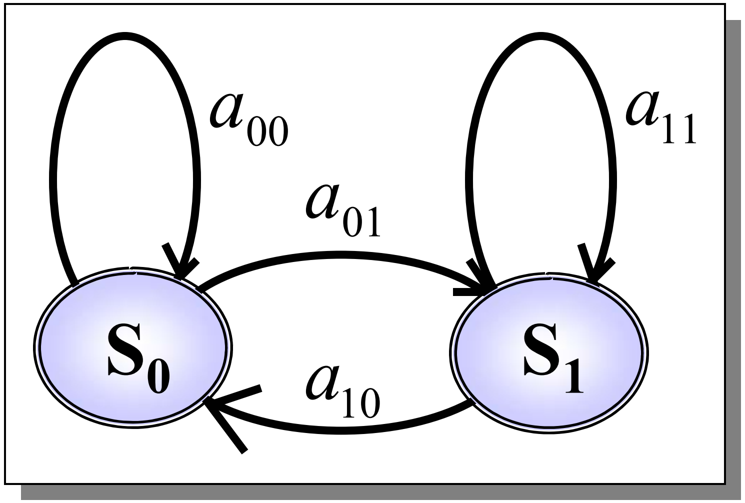
- L. R. Rabiner, "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition," Proc. IEEE, Vol. 77, No. 2, pp. 257-286, February, 1989.
- L.R. Rabiner & B. W. Juang, Fundamentals of Speech Recognition, Prentice-Hall, ISBN 0-13 015157-2, 1993 (see chapter 6)
- The Cambridge HTK Toolkit has a user manual called the "HTK Book". This is an excellent resource and covers many of the equations related to HMM modeling

Discrete-Time Markov Process

- **Characterized by:**
  - ❑ A set of N states

$$S = \{S_0, S_1, \cdots S_N\}$$

  - ❑ Transition Probabilities a_{ij}



- **First-order Markov Chain**
  - ❑ Current system state only depends on previous state
  - ❑ Let q_t be the system state at time t.
  - ❑ Transition probability depends only on previous state

$$P(q_t = S_j \mid q_{t-1} = S_i, q_{t-2} = S_k \cdots) = P(q_t = S_j \mid q_{t-1} = S_i)$$
$$a_{ij} = P(q_t = S_j \mid q_{t-1} = S_i)$$

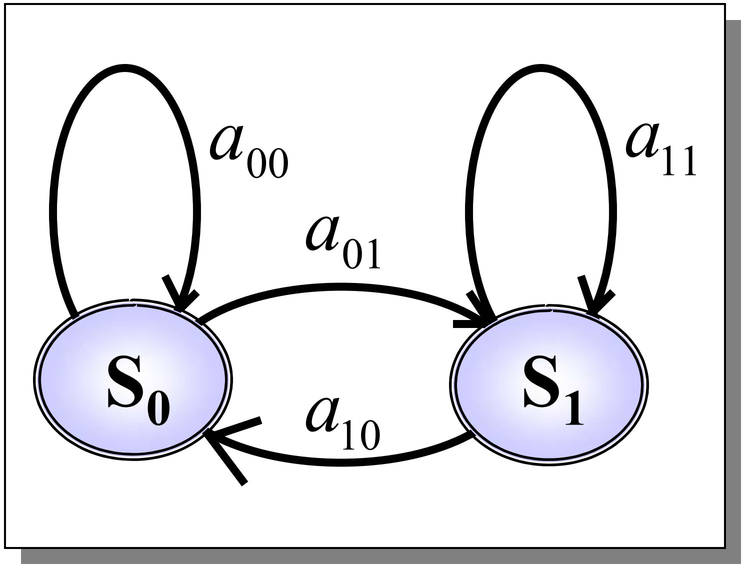
Discrete-Time Markov Process

- **Properties:**

$$A = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}$$

$$a_{ij} \geq 0 \quad \forall \, i, j$$

$$\sum_{j=0}^{N} a_{ij} = 1 \quad \forall \, i$$

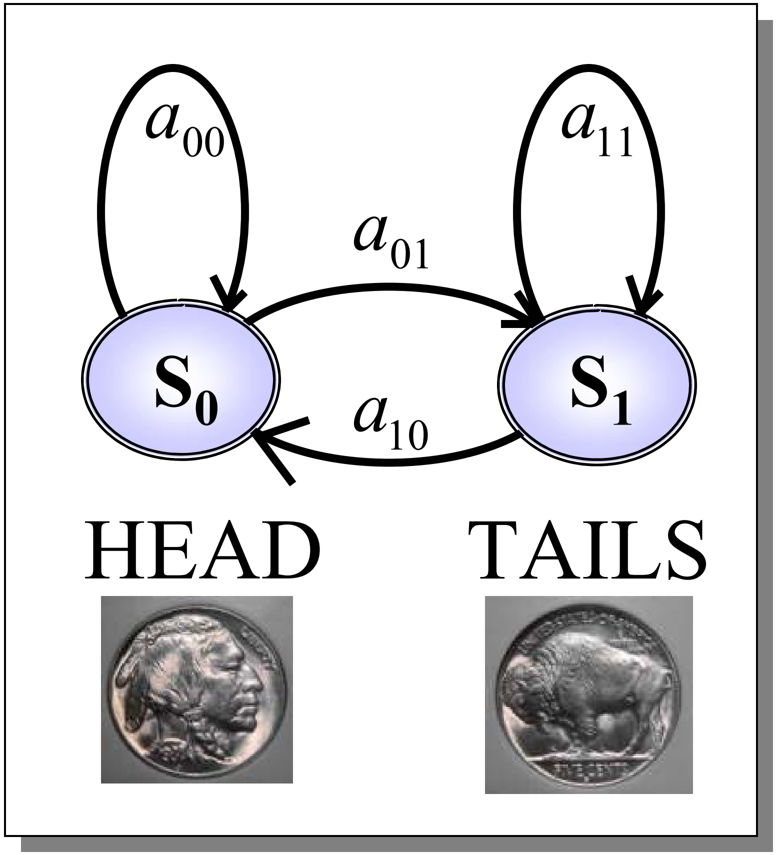


- **"Observable Markov Model" : output of the process is a set of states.**

Example 1: Single Fair Coin
Observable Markov Process

- **Outcomes**
  - Head (State 0)
  - Tail (State 1)
- **Observed outcomes uniquely define state sequence**
  - e.g., HHHTTTHHTT → S=0001110011
- **Transition Probabilities,**

$$A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$



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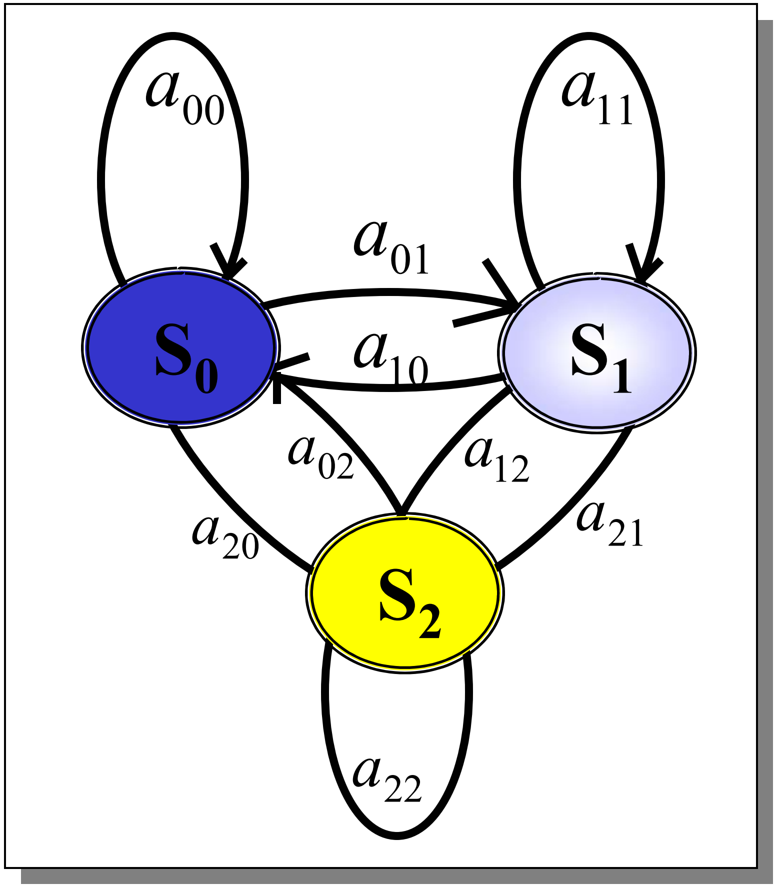
Example 2: Observable Markov Model of Weather

- **States:**
  - S_0 : rainy
  - S_1 : cloudy
  - S_2 : sunny
- **With state transition probabilities,**

$$A = \{a_{ij}\} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$



Observable Markov Model of Weather

- **What is the probability that the weather for 8 consecutive days is "sun, sun, sun, rain, rain, sun, cloudy, sun"?**

- **Solution**
  - Observation sequence:
    O = {sun, sun, sun, rain, rain, sun, cloudy, sun"}
  - Corresponds to state sequence, S = { 2, 2, 2, 0, 0, 2, 1, 2}
  - Want to determine, P(O | model)
  - P(O | model) = P (S={2,2,2,0,0,2,1,2} | model)

Observable Markov Model of Weather

π_i : initial state probability

$\pi_i = P(q_1 = i)$

$$A = \{a_{ij}\} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

$$P(O \mid \text{model}) = P(S = \{2,2,2,0,0,2,1,2\} \mid \text{model})$$

$$= P(q_1 = 2) P(q_2 = 2 \mid q_1 = 2) \cdots P(q_8 = 2 \mid q_7 = 1)$$

$$= \pi_2 \cdot a_{22} \cdot a_{22} \cdot a_{20} \cdot a_{00} \cdot a_{02} \cdot a_{21} \cdot a_{12}$$

$$= \pi_2 \cdot (0.8)^2 (0.1) \cdot (0.4) \cdot (0.3) \cdot (0.1) \cdot (0.2)$$

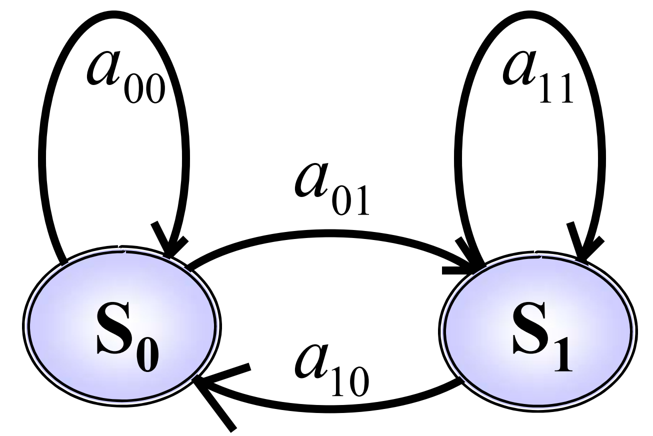
“Hidden” Markov Models

- Observations are a probabilistic function of state
- Underlying state sequence is not observable (hidden)
- First-order assumption
- Output independence: observations are dependent only on the state that generated them, not on eachother.

Example: 2-Coins, Observable Markov Process

- **Observations**
  - Head
  - Tail
- **Observed outcomes do not uniquely define state sequence**
- **Transition Probabilities,**

$$A = \begin{bmatrix} a_{00} & 1 - a_{00} \\ 1 - a_{11} & a_{11} \end{bmatrix}$$



$$P(H) = P_1 \qquad P(H) = P_2$$
$$P(T) = 1 - P_1 \qquad P(T) = 1 - P_2$$

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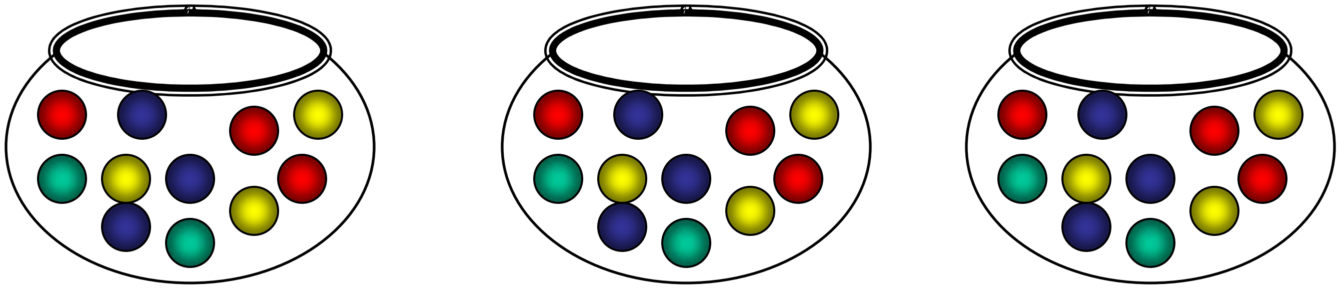
Urn-and-Ball Illustration



<u>A Genie plays a game with a blind observer</u>

1. **Genie initially selects an Urn at random**
2. **Genie picks a colored ball, tells blind observer the color**
3. **Genie puts ball back in Urn**
4. **Genie moves to next Urn based on random selection procedure from current Urn.**
5. **Steps 2-4 repeated**

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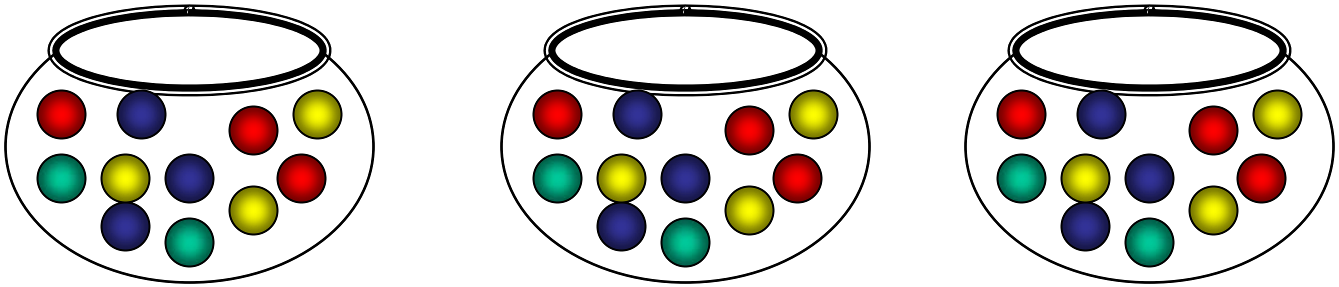
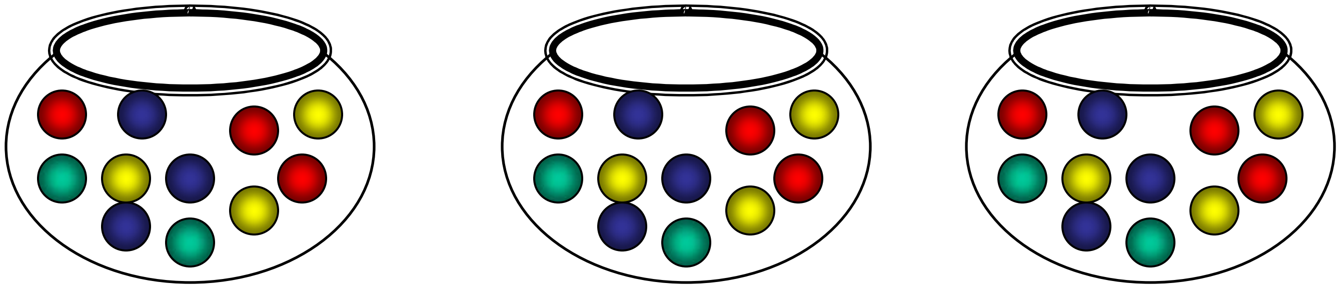
Urn-and-Ball Illustration



- **Observations:**
  - ❑ The color sequence of the balls
- **States:**
  - ❑ The identity of the urn
- **State-Transition:**
  - ❑ The process of selecting the urns

Urn-and-Ball Illustration



$P(R) = 0.20$	$P(R) = 0.45$	$P(R) = 0.15$
$P(G) = 0.30$	$P(G) = 0.15$	$P(G) = 0.70$
$P(B) = 0.10$	$P(B) = 0.20$	$P(B) = 0.10$
$P(Y) = 0.60$	$P(Y) = 0.20$	$P(Y) = 0.05$

- **Observation sequence: R B Y Y G B Y G R …**
  - ❑ Individual colors (observations) don't reveal urn (state)

Discrete Symbol Observation HMM

- **A set of N states**

$$S = \{S_0, S_1, \cdots S_N\}$$

- **Transition Probabilities**

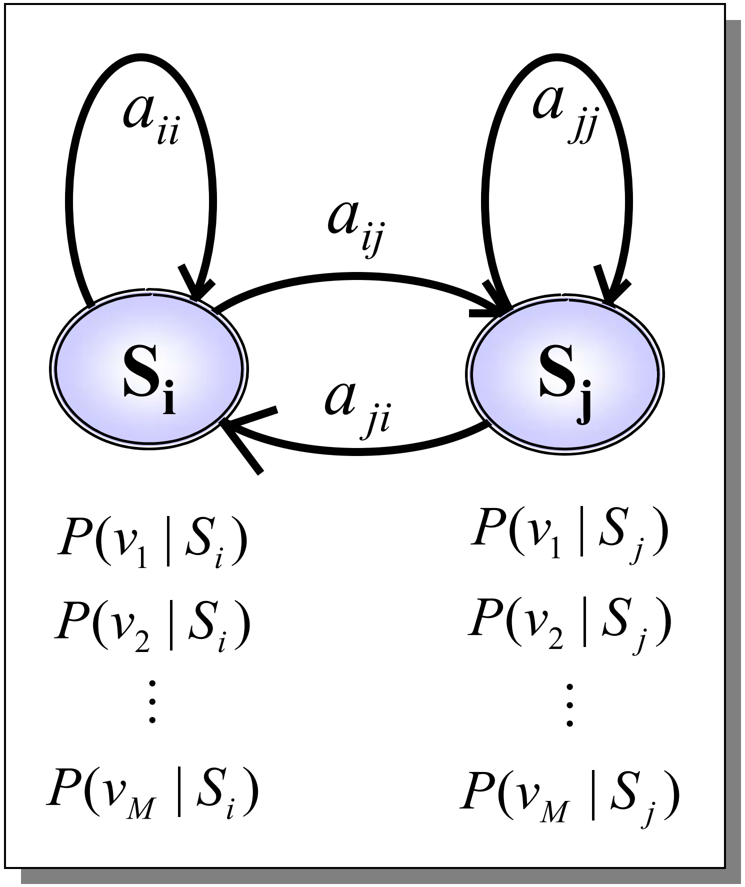
$$a_{ij} = P(q_t = S_j \mid q_{t-1} = S_i)$$

- **A set of M observation symbols**

$$V = \{v_1, v_2, \cdots v_M\}$$

- **Probability Distribution (for state j, symbol k)**

$$b_j(k) = P(o_t = v_k \mid q_t = j)$$



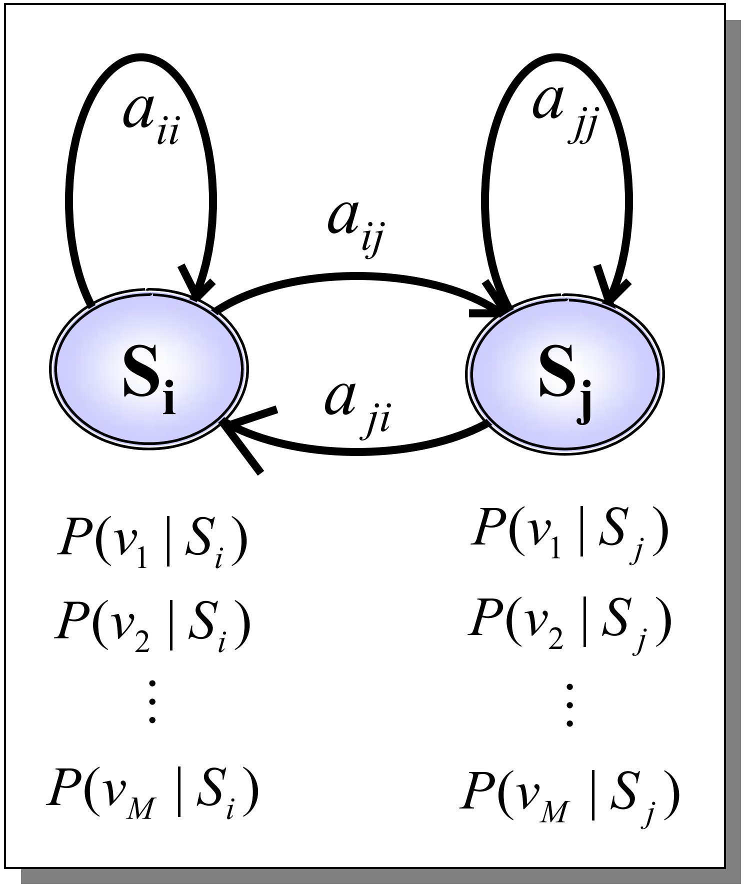
Discrete Symbol Observation HMM

- **Also characterized by:**
  - ❑ An initial state distribution

$$\pi = \{\pi_i\} = P(q_1 = i)$$

- **Specification thus requires**
  - ❑ 2 model parameters, N and M
  - ❑ Specification of M symbols
  - ❑ 3 probability measures A,B,π

$$\lambda = (A, B, \pi)$$

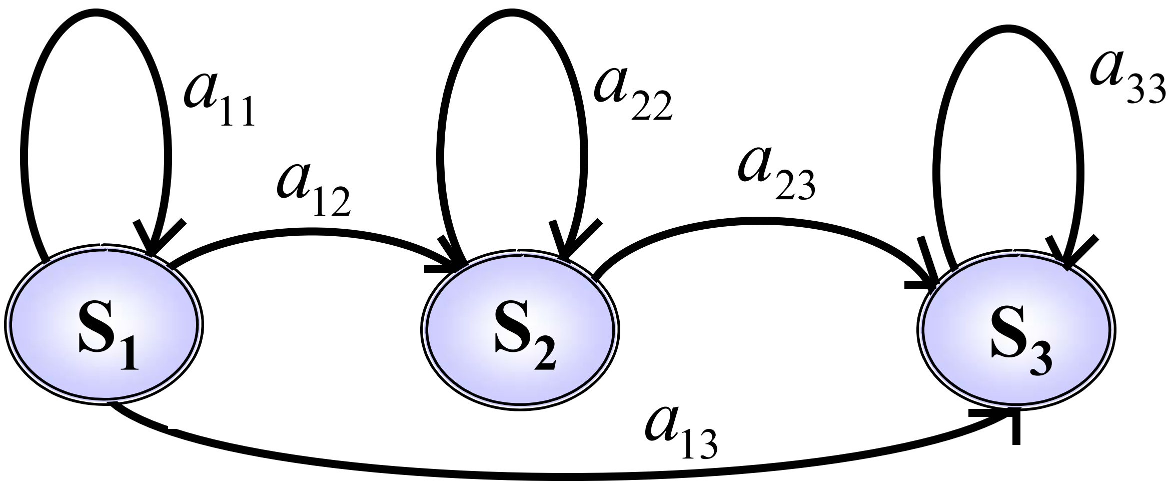


Left-to-Right HMMs

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{ij} = 0 \qquad j < i$$

$$\pi_i = \begin{cases} 0, & i \neq 1 \\ 1, & i = 1 \end{cases}$$



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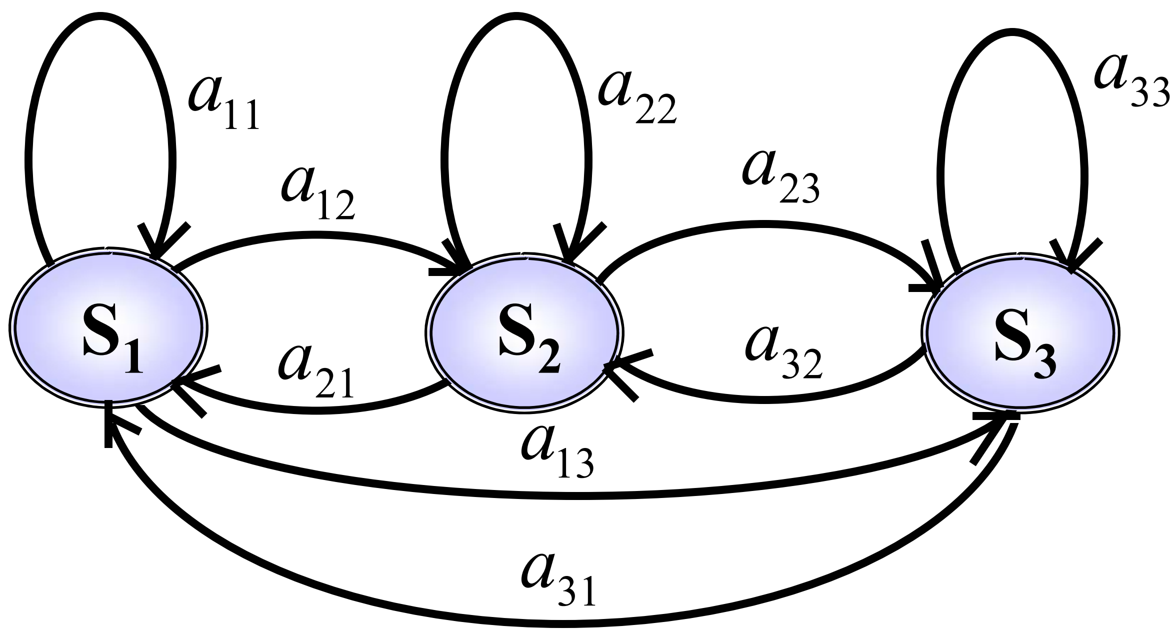
Ergodic HMMs

$$a_{ij} > 0 \qquad \forall i, \forall j$$



HMM as a Symbol Generator

- **Given parameters N, M, A, B, and π, HMMs can generate an observation sequence,**

$$\mathbf{O} = \{\mathbf{o}_1, \mathbf{o}_2, \cdots, \mathbf{o}_T\}$$

- **1. Choose initial state** $q_1 = i$ **from init. state distribution π**
- **2. Set** $t = 1$
- **3. Choose** $\mathbf{o}_t = \mathbf{v}_k$ **according to distribution** $b_i(k)$
- **4. Transition to set state** $q_{t+1} = j$ **according to state-transition probability distribution** a_{ij}
- **5. Set** $t = t + 1$ **go to step 3**

HMM as a Symbol Generator

- **Example output from symbol generation**

Time, t	1	2	3	4	5	6	...	T
State	q_1	q_2	q_3	q_4	q_5	q_6		q_T
Observation	$\mathbf{o}_1$	$\mathbf{o}_2$	$\mathbf{o}_3$	$\mathbf{o}_4$	$\mathbf{o}_5$	$\mathbf{o}_6$		$\mathbf{o}_T$

- **We can think of the HMM as generating the observation sequence as it transitions from state to state.**

Three Interesting Problems

- **Problem 1: Scoring & Evaluation**
  - How to efficiently compute the probability of an observation sequence (O) given a model (λ)? → $P(O|\lambda)$
- **Problem 2: Decoding**
  - Given an observation sequence (O) and a model (λ), how do we determine the corresponding state-sequence (q) that "best explains" how the observations were generated?
- **Problem 3: Training**
  - How to adjust model parameters ($\lambda = \{A, B, \pi\}$) to maximize probability of generating a given observation sequence? → maximize $P(O|\lambda)$.

Problem 1: Scoring and Evaluation

- Given an observation sequence,

$$\mathbf{O} = \{\mathbf{o}_1, \mathbf{o}_2, \cdots, \mathbf{o}_T\}$$

- Want to compute probability of generating it:

$$P(\mathbf{O} \mid \lambda)$$

- Let's assume a particular sequence of states,

$$q = \{q_1, q_2, \cdots, q_T\}$$

Problem 1: Scoring and Evaluation

- **Using the chain rule we can decompose the problem by summing over all possible state sequences,**

$$\mathrm{P}(\mathbf{O} \mid \lambda) = \sum_{\text{all q}} \mathrm{P}(\mathbf{O} \mid q, \lambda)\mathrm{P}(q \mid \lambda)$$

- **The first term relates to the likelihood of generating the observed symbol sequence given the assumed state sequence.**

- **The second term relates to how likely the system is to step through those sequence of states.**

Problem 1: Scoring and Evaluation

- **Probability of the observation sequence given the state sequence,**

$$P(\mathbf{O} \mid q, \lambda) = \prod_{t=1}^{T} p(\mathbf{o}_t \mid q_t, \lambda)$$

$$= b_{q_1}(\mathbf{o}_1) \cdot b_{q_2}(\mathbf{o}_2) \cdots b_{q_T}(\mathbf{o}_T)$$

- **Probability of the state sequence,**

$$P(q \mid \lambda) = \pi_{q_1}(a_{q_1 q_2}) \cdot (a_{q_2 q_3}) \cdots (a_{q_{T-1} q_T})$$

Problem 1: Scoring and Evaluation

- **Using the chain rule,**

$$\mathrm{P}(\mathbf{O} \mid \lambda) = \sum_{\text{all q}} \mathrm{P}(\mathbf{O} \mid q, \lambda)\mathrm{P}(q \mid \lambda)$$

$$= \sum_{\text{all q}} \pi_{q_1} b_{q_1}(\mathrm{o}_1) a_{q_1 q_2} b_{q_2}(\mathrm{o}_2) \cdots a_{q_{T-1} q_T} b_{q_T}(\mathrm{o}_T)$$

- **This is not practical to compute. For N states, T observations, the number of state sequences is:**

$$\mathrm{O}(2T * N^T)$$

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Forward Algorithm

- **Definition:** $\alpha_t(i) = P(\mathbf{o}_1\mathbf{o}_2 \ldots \mathbf{o}_t, q_t = i \mid \lambda)$

  (Probability of seeing observations o_1 to o_t and ending at state *i* given HMM λ)

1. **Initialization** $\alpha_0(i) = \pi_i$
2. **Induction** $\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right] b_j(\mathbf{o}_{t+1})$
3. **Termination** $P(\mathbf{O} \mid \lambda) = \sum_{i=1}^{N} \alpha_T(i)$

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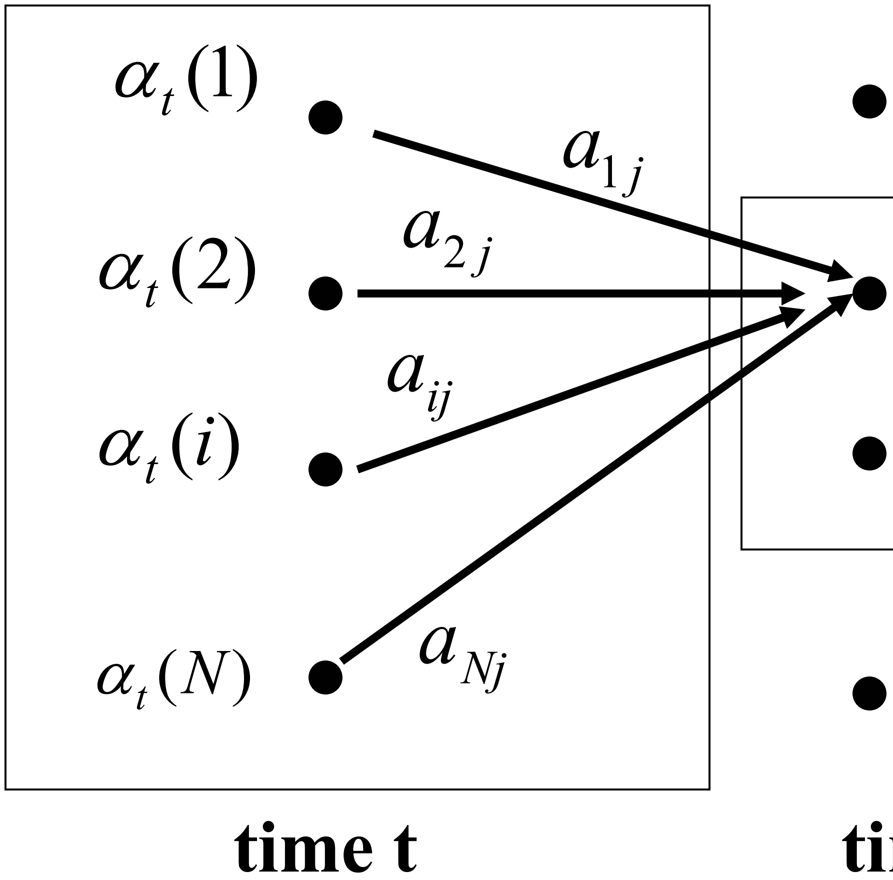
Forward Algorithm Illustration



$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij}\right] b_j(\mathbf{o}_{t+1})$$

$O(T * N^2)$

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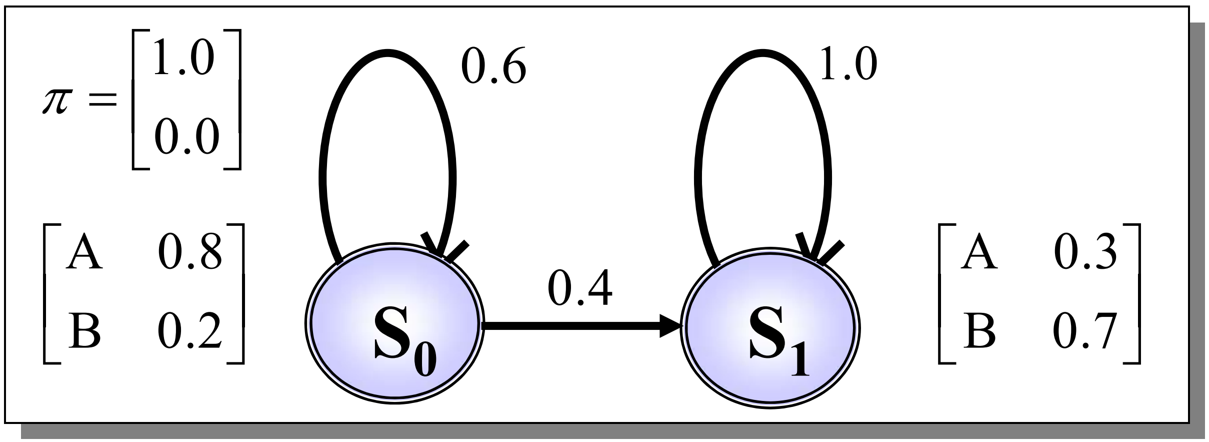
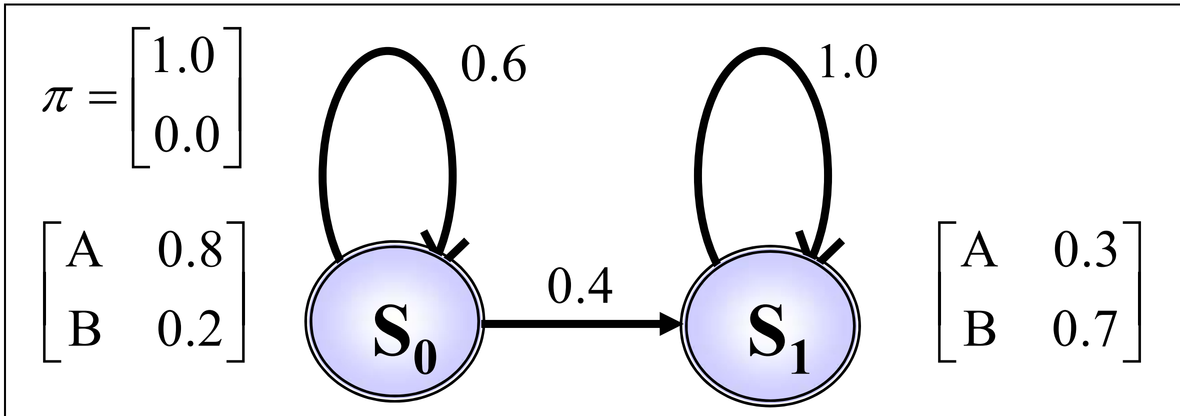
Forward Algorithm Example

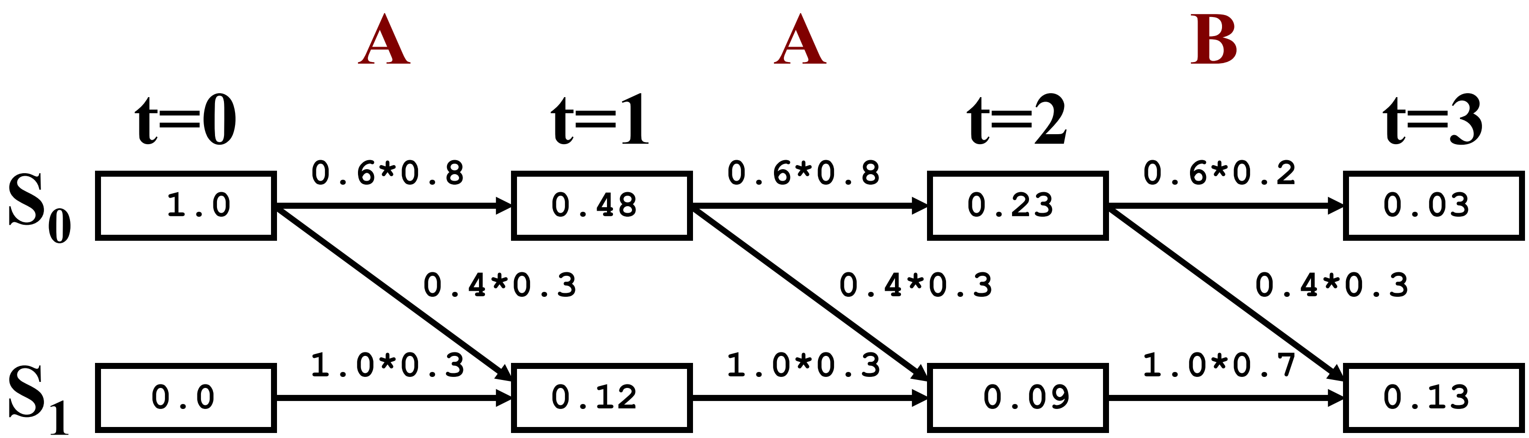


- **Given the above HMM with discrete observations “A” and “B”, what is the probability of generating the sequence “O = {A,A,B}”?**
- **In other words, find P(O={A,A,B} | λ)**

Forward Algorithm Example



Answer:
$P(O|\lambda)$=
0.03+0.13
= 0.16



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Backward Algorithm

- **Definition:** $\beta_t(i) = P(\mathbf{o}_{t+1}\mathbf{o}_{t+2}\ldots\mathbf{o}_T, q_t = i \mid \lambda)$
- **Probability of observation sequence o_{t+1} to o_T given state i at time t and HMM λ**
- **Initialization** $\beta_T(i) = 1$
- **Induction** $$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(\mathbf{o}_{t+1})\beta_{t+1}(j)$$

$t = T-1, T-2, \ldots, 1$

$1 \le i \le N$

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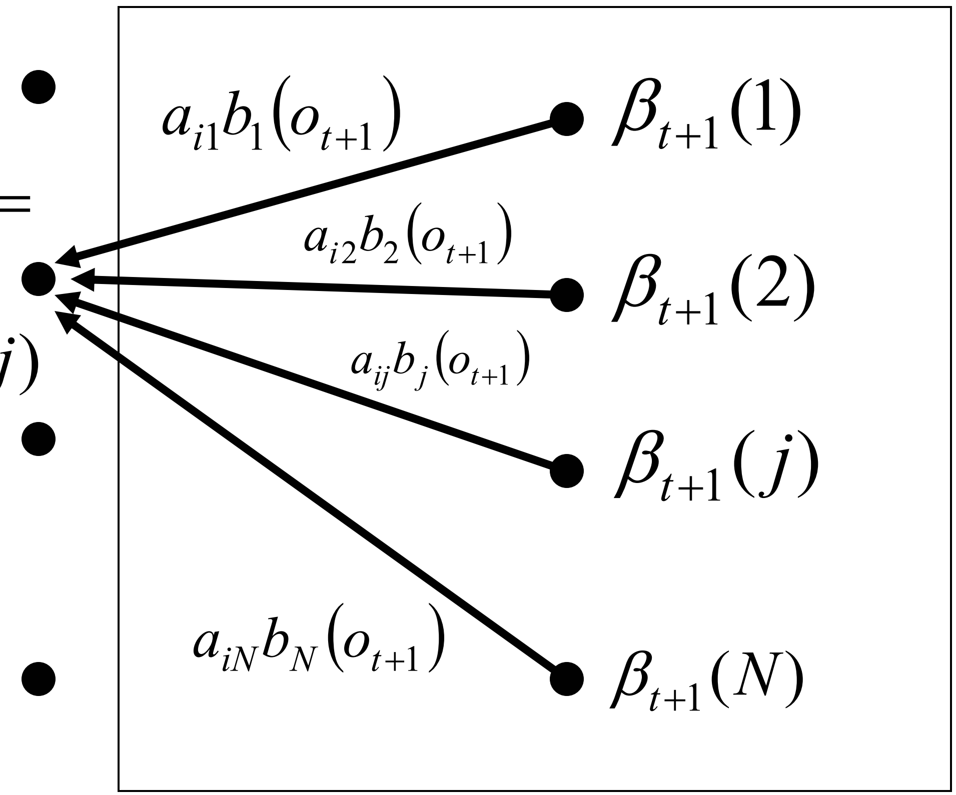
Backward Algorithm Illustration

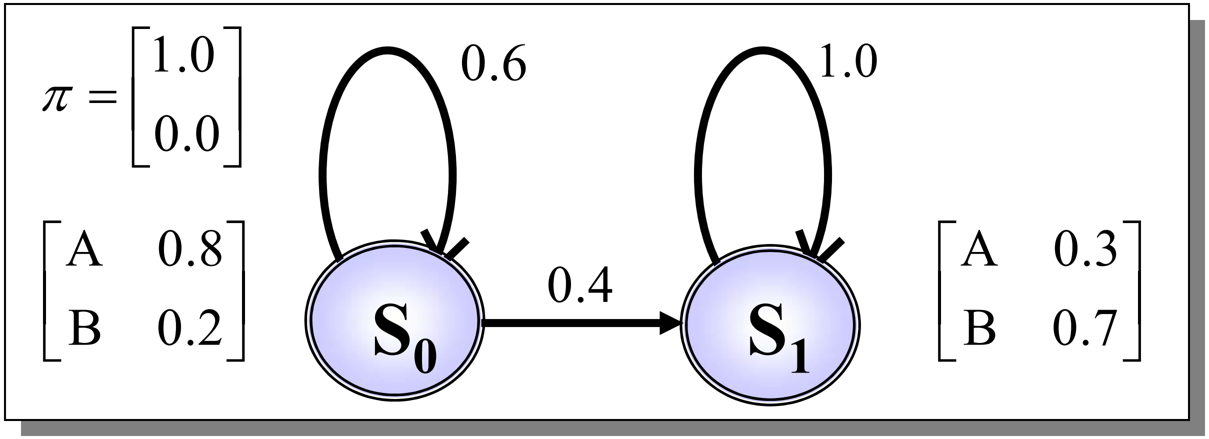
$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(\mathbf{o}_{t+1}) \beta_{t+1}(j)$$

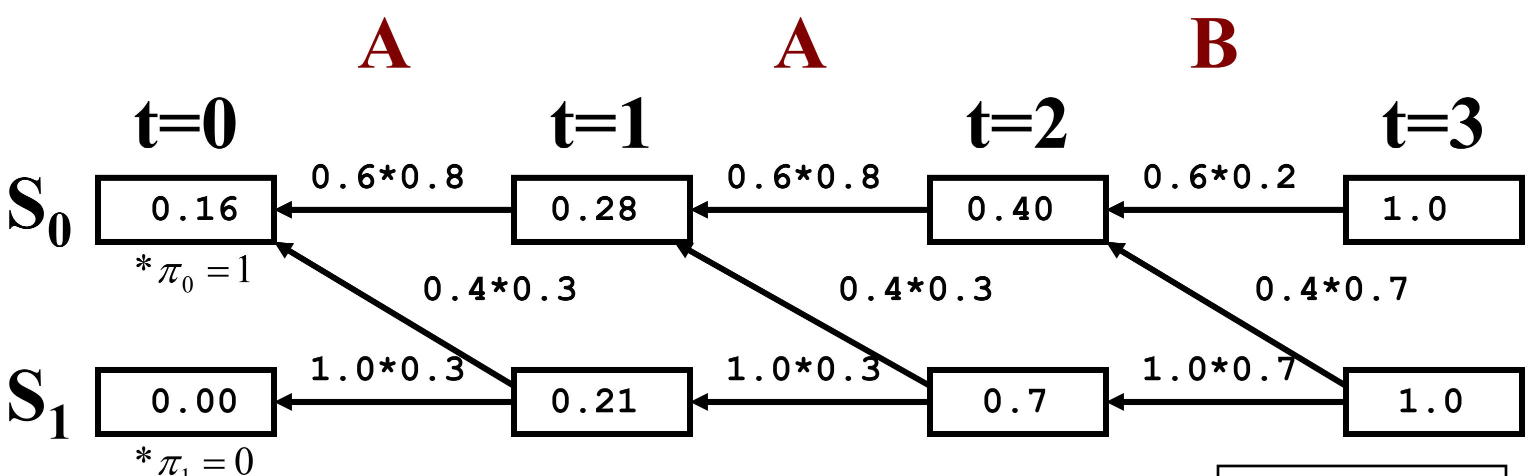


time t **time t+1**

$O(T * N^2)$

Backward Algorithm Example





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Problem 1: Scoring and Evaluation

- **In fact, there are 2 ways to compute** $P(\mathbf{O} \mid \lambda)$

$$P(\mathbf{O} \mid \lambda) = \sum_{i=1}^{N} \alpha_T(i) \qquad \textbf{Forward Algorithm}$$

$$P(\mathbf{O} \mid \lambda) = \sum_{i=1}^{N} \pi_i \beta_0(i) \qquad \textbf{Backward Algorithm}$$

- **Computation of each term is** $\mathrm{O}(N^2 T)$

Problem 2: Decoding

- **Given an observation sequence,**

$$\mathbf{O} = \{\mathbf{o}_1, \mathbf{o}_2, \cdots, \mathbf{o}_T\}$$

- **Find the single best sequence of states,**

$$q = \{q_1, q_2, \cdots, q_T\}$$

- **Which maximizes,**

$$P(\mathbf{O}, q \mid \lambda)$$

Solution: Viterbi Algorithm

- **Define:**

$$\delta_t(i) = \max_{q_1 q_2 \cdots q_{t-1}} P(q_1 q_2 \cdots q_{t-1,} q_t = i, \mathbf{o}_1 \mathbf{o}_2 \ldots \mathbf{o}_t \mid \lambda)$$

"Highest probable state sequence that accounts for observations *1* through *t-1* and ends in state *i* and time *t*".

$$\delta_{t+1}(j) = [\max_i \delta_t(i) a_{ij}] \cdot b_j\left(o_{t+1}\right)$$

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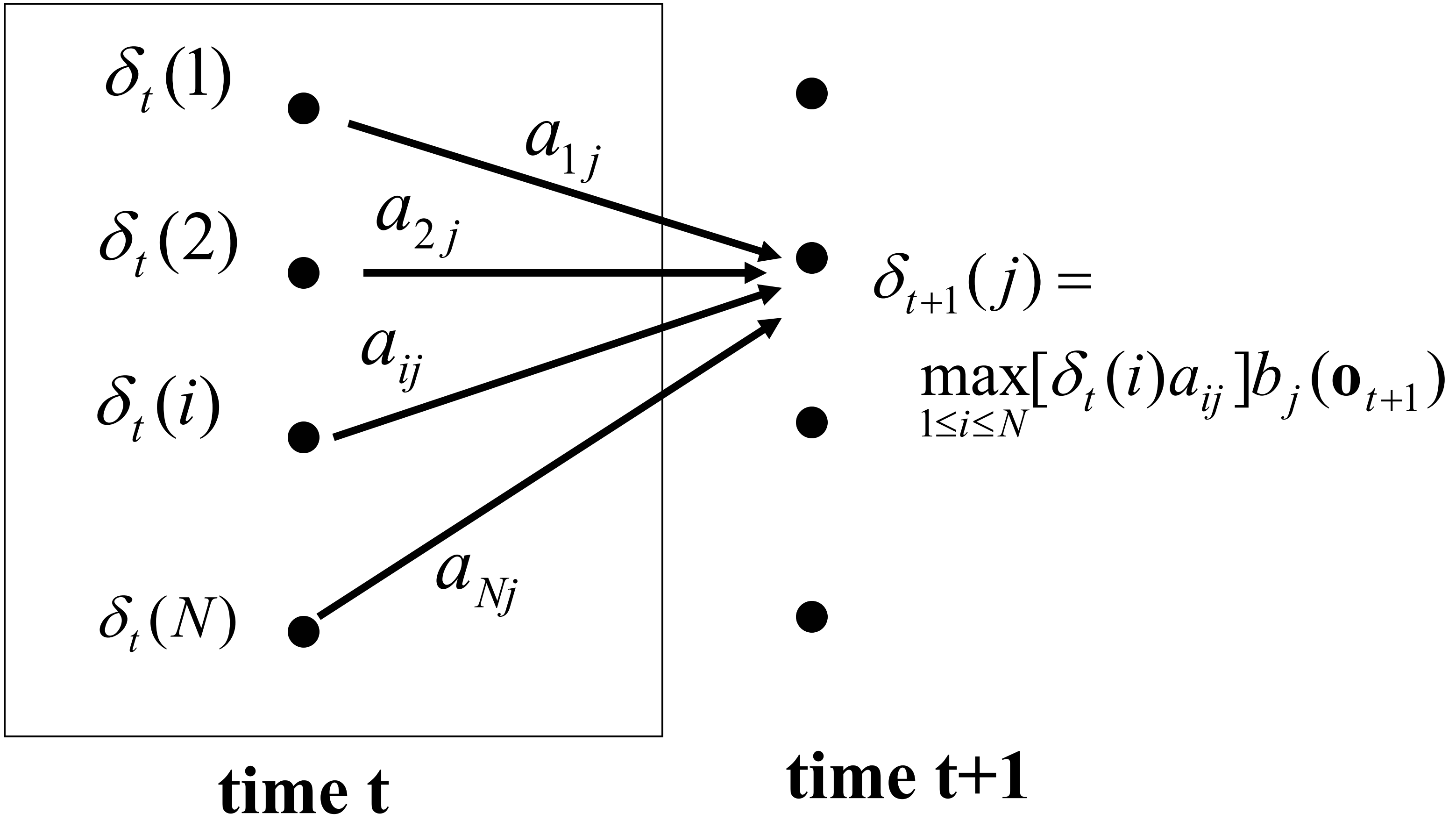
Viterbi Algorithm Illustration



Viterbi Algorithm

1. Initialization $\delta_1(i) = \pi_i b_i(\mathbf{o}_1)$ $\psi_1(i) = 0$

2. Recursion

$$\delta_t(j) = \max_{1 \le i \le N} [\delta_{t-1}(i) a_{ij}] b_j(\mathbf{o}_t)$$

$$\psi_t(j) = \arg\max_{1 \le i \le N} [\delta_{t-1}(i) a_{ij}]$$

3. Termination $P^* = \max_{1 \le i \le N} [\delta_T(i)]$ $q_T^* = \arg\max_{1 \le i \le N} [\delta_T(i)]$

4. Path Back trace $q_t^* = \psi_{t+1}(q_{t+1}^*)$

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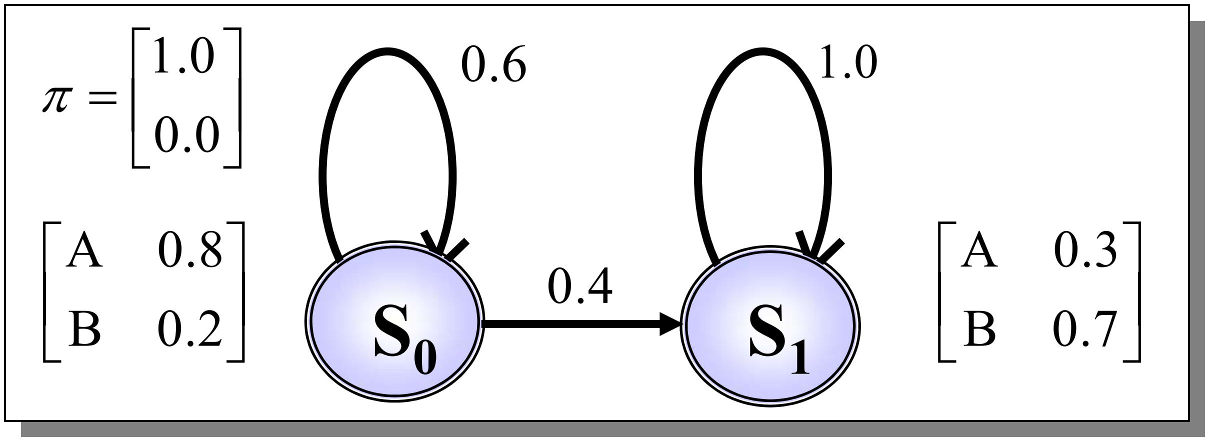
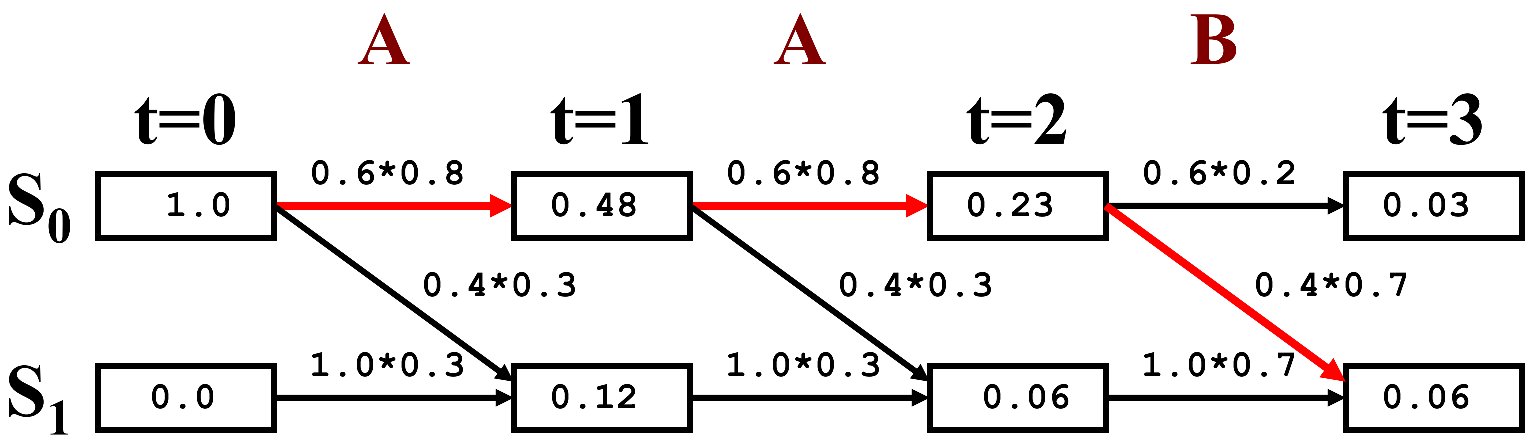
Viterbi Algorithm Illustration





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Viterbi Algorithm in Log-Domain

- **Due to numerical underflow issues, it is often common to conduct the Viterbi algorithm in the log-domain,**

$$\tilde{\pi}_i = \log(\pi_i)$$

$$\tilde{b}_j(o_t) = \log(b_j(o_t))$$

$$\tilde{a}_{ij} = \log(a_{ij})$$

- **Viterbi search in speech recognition systems is implemented in the log-domain for example.**

Viterbi Algorithm in Log-Domain

1. Initialization $\tilde{\delta}_1(i) = \tilde{\pi}_i + \tilde{b}_i(\mathbf{o}_1) \quad \psi_1(i) = 0$

2. Recursion

$$\delta_t(j) = \max_{1 \le i \le N} [\tilde{\delta}_{t-1}(i) + \tilde{a}_{ij}] + \tilde{b}_j(\mathbf{o}_t)$$

$$\psi_t(j) = \arg\max_{1 \le i \le N} [\tilde{\delta}_{t-1}(i) + \tilde{a}_{ij}]$$

3. Termination $\tilde{P}^* = \max_{1 \le i \le N} \left[\tilde{\delta}_T(i)\right] \quad q_T^* = \arg\max_{1 \le i \le N} \left[\tilde{\delta}_T(i)\right]$

4. Path Back trace $q_t^* = \psi_{t+1}(q_{t+1}^*)$

Problem 3: Training

- **Train parameters of HMM**
  - Tune λ to maximize $P(O|\lambda)$
  - No efficient algorithm for global optimum
  - Efficient iterative algorithm finds a local optimum

- **(Baum-Welch) Forward-Backward Algorithm**
  - Compute probabilities using current model λ
  - Refine $\lambda \rightarrow \overline{\lambda}$ based on computed values
  - Uses α and β from forward and backward algorithms

Forward-Backward Algorithm

- **Define:**

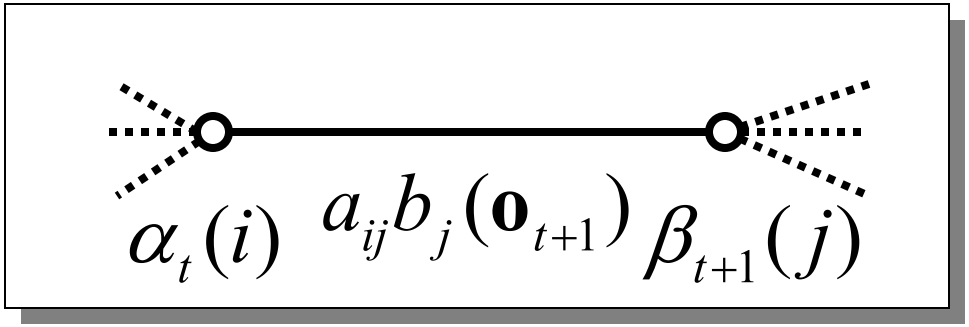
$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j \mid \mathbf{O}, \lambda)$$
$$= \frac{P(q_t = i, q_{t+1} = j, \mathbf{O} \mid \lambda)}{P(\mathbf{O} \mid \lambda)}$$

"Probability of being in state *i* at time *t* and state *j* at time *t+1* given the model and observation sequence"

Forward-Backward Algorithm

$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j \mid \mathbf{O}, \lambda)$$

$$= \frac{\alpha_t(i) a_{ij} b_j(\mathbf{o}_{t+1}) \beta_{t+1}(j)}{P(\mathbf{O} \mid \lambda)}$$



Forward-Backward Algorithm

$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j \mid \mathbf{O}, \lambda)$$

$$= \frac{\alpha_t(i) a_{ij} b_j(\mathbf{o}_{t+1}) \beta_{t+1}(j)}{P(\mathbf{O} \mid \lambda)}$$

$$= \frac{\alpha_t(i) a_{ij} b_j(\mathbf{o}_{t+1}) \beta_{t+1}(j)}{\sum_{i=1}^{N} \alpha_t(i) \beta_t(i)}$$

Forward-Backward Algorithm

$$\gamma_t(i) = \sum_{j=1}^{N} \xi_t(i, j)$$

Probability of being in state *i* at time *t*

$$\sum_{t=1}^{T-1} \gamma_t(i) =$$

Expected number of transitions from state *i* in *O.*

$$\sum_{t=1}^{T-1} \xi_t(i, j) =$$

Expected number of transitions from state *i* to state *j* in O.

Forward-Backward Algorithm

- **Using these definitions, we can compute the initial state occupancy probability,**

$$\bar{\pi}_i = \text{expected number of times in state } i \text{ at time } t = 1$$

$$= \gamma_1(i)$$

Forward-Backward Algorithm

$$\bar{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

$$= \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{j=1}^{N} \xi_t(i,j)} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

Forward-Backward Algorithm

$$\bar{b}_j(k) = \frac{\text{expected number of times in state } j \text{ and observing kth symbol}}{\text{expected number times in state } j}$$

$$= \frac{\sum_{\substack{t=1 \\ \mathbf{o}_t = v_k}}^{T} \sum_{j=1}^{N} \xi_t(j,j)}{\sum_{t=1}^{T} \sum_{j=1}^{N} \xi_t(j,j)} = \frac{\sum_{\substack{t=1 \\ \mathbf{o}_t = v_k}}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)} = \frac{\sum_{\substack{t=1 \\ \mathbf{o}_t = v_k}}^{T} \alpha_t(j)\beta_t(j)}{\sum_{t=1}^{T} \alpha_t(j)\beta_t(j)}$$

Forward-Backward Algorithm

1. Initialize λ=(A,B,π)

2. Compute α, β, and ξ

3. Estimate $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$

4. Replace λ with $\bar{\lambda}$

5. If not converged go to 2

It can be shown That

$P(O|\bar{\lambda}) > P(O|\lambda)$

Unless

$\bar{\lambda} = \lambda$

FB Algorithm Satisfies Constraints:

$$\sum_{i=1}^{N} \overline{\pi}_i = 1$$

$$\sum_{j=1}^{N} \overline{a}_{ij} = 1 \qquad 1 \le i \le N$$

$$\sum_{k=1}^{M} \overline{b}_j(k) = 1 \qquad 1 \le j \le N$$

Continuous Observation Densities

- Discussions up until this point have considered observations characterized by discrete symbols
- Often, we work with continuous valued data
- Can convert continuous variables to discrete symbols using a Vector Quantizer (VQ) codebook [with information loss]
- How to model continuous observations directly?

Mixture Gaussian PDFs

- **Mixture Gaussian PDF with M components,**

$$b_j(\mathbf{o_t}) = \sum_{k=1}^{M} c_{jk} \mathrm{N}(\mathbf{o_t}, \mu_{jk}, \Sigma_{jk})$$

- **Constraints on the mixture weights,**

$$\sum_{k=1}^{M} c_{jk} = 1$$

$$c_{jk} \geq 0, \qquad 1 \leq \mathrm{k} \leq \mathrm{M}$$

Definition

- **Probability of being in state *j* at time *t* with the *k*th mixture component accounting for $\mathbf{o}_t$:**

$$\gamma_t(j,k) = \left[\frac{\alpha_t(j)\beta_t(j)}{\sum_{j=1}^{N} \alpha_t(j)\beta_t(j)} \right] \left[\frac{c_{jk} \mathrm{N}(\mathbf{o}_t, \mu_{jk}, \Sigma_{jk})}{\sum_{m=1}^{M} c_{jm} \mathrm{N}(\mathbf{o}_t, \mu_{jm}, \Sigma_{jm})} \right]$$

Resulting Equations for Mixture Weight & Mean Update

$$\bar{c}_{jk} = \frac{\sum_{t=1}^{T} \gamma_t(j,k)}{\sum_{t=1}^{T} \sum_{k=1}^{M} \gamma_t(j,k)}$$

$$\bar{\mu}_{jk} = \frac{\sum_{t=1}^{T} \gamma_t(j,k) \cdot \mathbf{o}_t}{\sum_{t=1}^{T} \sum_{k=1}^{M} \gamma_t(j,k)}$$

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Resulting Equations for Transition Matrix & Variance Update

- **Transition probability a_{ij} same as case for discrete symbols. Covariance Matrix:**

$$\overline{\Sigma}_{jk} = \frac{\sum_{t=1}^{T} \gamma_t(j,k) \cdot \left(\mathbf{o}_t - \overline{\mu}_{jk}\right)\left(\mathbf{o}_t - \overline{\mu}_{jk}\right)'}{\sum_{t=1}^{T} \sum_{k=1}^{M} \gamma_t(j,k)}$$

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Estimation from Multiple Observation Sequences

- **We estimate HMM parameters from multiple examples of speech:**
  - Collected from different speakers
  - In different contexts
- **We attempt to model variability in producing each sound unit by estimating parameters from large corpora of speech data.**

Multiple Observations

- **Assume K training patterns (observation sequences),**

$$\mathbf{O} = \left[\mathbf{O}^{(1)}, \mathbf{O}^{(2)}, \ldots, \mathbf{O}^{(K)}\right]$$

- **Where,**

$$\mathbf{O}^{(\mathbf{k})} = \left\{\mathbf{o}_{\mathbf{1}}^{(\mathbf{k})}, \mathbf{o}_{\mathbf{1}}^{(\mathbf{k})}, \ldots, \mathbf{o}_{\mathbf{T_k}}^{(\mathbf{k})}\right\}$$

- **And,**

$$\mathrm{P}_k = P(\mathbf{O}^{(\mathrm{k})} \mid \lambda)$$

Transition Probabilities with Multiple Observations

$$\bar{a}_{ij} = \frac{\sum_{k=1}^{K} \frac{1}{P_k} \sum_{t=1}^{T_k - 1} \alpha_t^k(i) a_{ij} b_j(\mathbf{o}_{\mathbf{t+1}}^{(\mathbf{k})}) \beta_{t+1}^k(j)}{\sum_{k=1}^{K} \frac{1}{P_k} \sum_{t=1}^{T_k - 1} \alpha_t^k(i) \beta_t^k(i)}$$

Observation Probabilities with Multiple Observations

$$\bar{b}_j(l) = \frac{\displaystyle\sum_{k=1}^{K} \frac{1}{P_k} \sum_{\substack{t=1 \\ s.t. O_t = v_l}}^{T_k - 1} \alpha_t^k(i)\beta_t^k(i)}{\displaystyle\sum_{k=1}^{K} \frac{1}{P_k} \sum_{t=1}^{T_k - 1} \alpha_t^k(i)\beta_t^k(i)}$$

Single State HMM Example

- **How to estimate the parameters of a single-state HMM model with M component Gaussians?,**

$$b(\mathbf{o_t}) = \sum_{k=1}^{M} w_k b_k(\mathbf{o_t}, \mu_k, \Sigma_k)$$

$$= \sum_{k=1}^{M} \frac{w_k}{(2\pi)^{d/2} |\Sigma_k|^{1/2}} \exp\left(-\frac{1}{2} (o_t - u_k)' \Sigma_k^{-1} (o_t - u_k) \right)$$

Single State HMM Example

- Define probability of *t*th observation being generated by the *k*th mixture component,

$$P(k \mid o_t, \lambda) = \frac{w_k b_k(o_t)}{\sum_{k=1}^{M} w_k b_k(o_t)}$$

- Note that "k" (b_k) here refers to mixture component, not HMM state. We are assuming just 1 HMM state.

Single State HMM Update Equations

Updated (new) parameter estimates

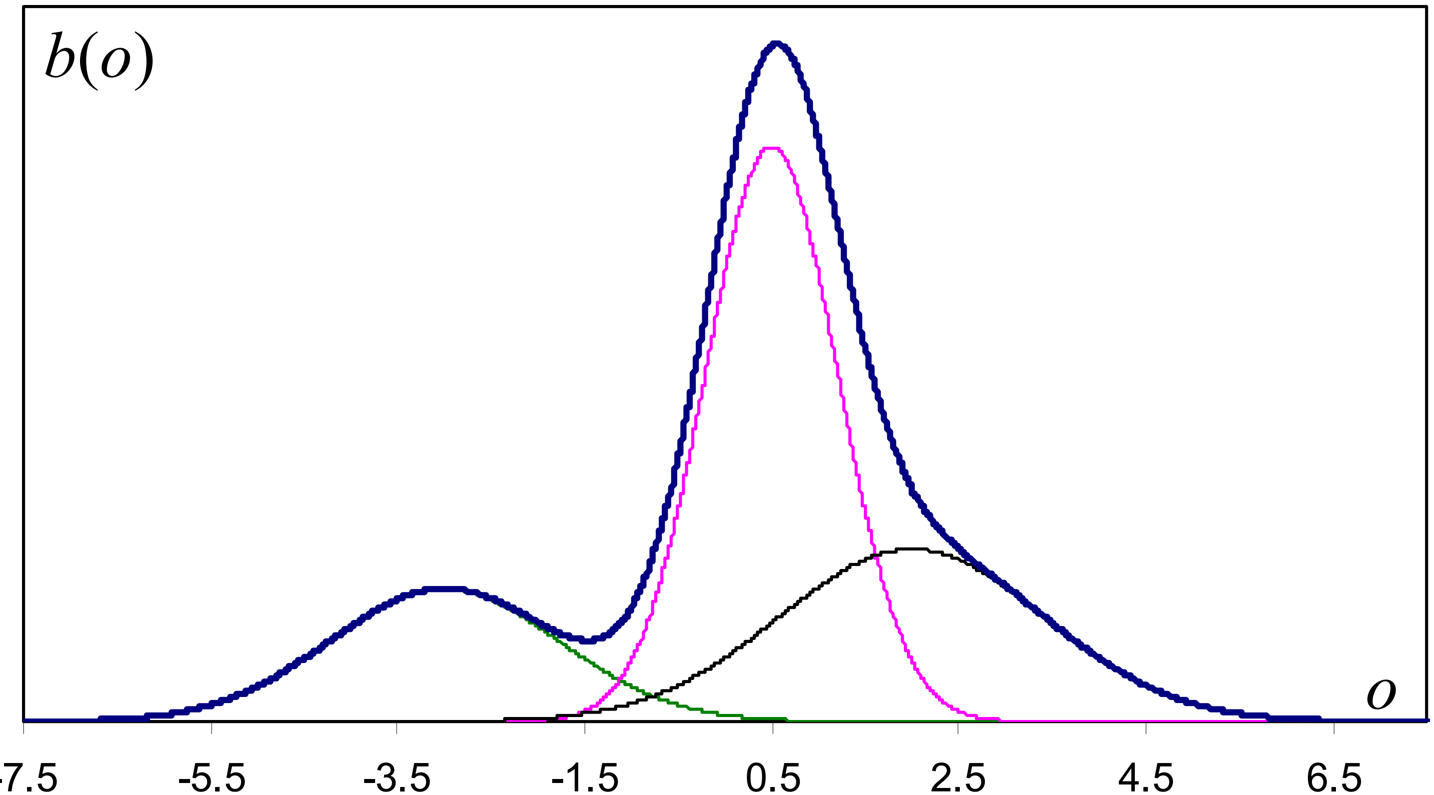
$$\boxed{\overline{w}_k} = \frac{1}{T}\sum_{t=1}^{T}\boxed{P(k \mid o_t, \lambda)}$$

This term is computed using model parameters from previous algorithm iteration.

$$\boxed{\overline{u}_k} = \frac{\sum_{t=1}^{T} P(k \mid o_t, \lambda)\cdot o_t}{\sum_{t=1}^{T} P(k \mid o_t, \lambda)}$$

$$\boxed{\overline{\Sigma}_k} = \frac{\sum_{t=1}^{T} P(k \mid o_t, \lambda)\cdot (o_t)^2}{\sum_{t=1}^{T} P(k \mid o_t, \lambda)} - (\overline{u}_k)^2$$

Mixture Gaussian (1-D case)



Example: 3 mixtures used to model underlying random process of 3 Gaussians

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Homework #3

- You will estimate the parameters of a single state multivariate HMM given a sequence of observations (O).
- Training parameters (observations) are MFCCs (13 dimensional) from 10 speakers
- Test parameters: from unknown speakers… which speaker most likely produced the sequence?

Homework #3

- **Can compute the probability of generating the observation sequence for each estimated single-state HMM:**

$$\log P(O \mid \lambda_s) = \sum_{t=1}^{T} \log b(o_t)$$

- **Find the model which has the maximum log-probability…**

HMM Assumptions

- **First-Order Markov Chain**
  - Probability of transitioning to a state only dependents on the current state
- **Time-independence of State Transitions**
  - Transitioning from state A to state B is independent of time
- **Observation Independence**
  - Observations don’t depend on each other, just on the states that generated them
- **(sometimes) Left-to-Right Topology**
  - As we will see, it is generally assumed that a left-to-right HMM is used to model speech units (phonemes)

Important Concepts from Today

- **Do you understand the basic idea of an HMM?**
- **Could you implement the Viterbi Algorithm if you had to?**
- **Can you estimate at least the parameters of a single-state HMM (mixture-Gaussians)?**
- **What assumptions do HMMs impose on the data being modeled?**

Next Week

- **How to use HMMs for modeling speech units?**
- **Consider major strategies for acoustic training**
- **How to model context dependencies using HMMs**
- **Some practical notes on HMM training for speech recognition**