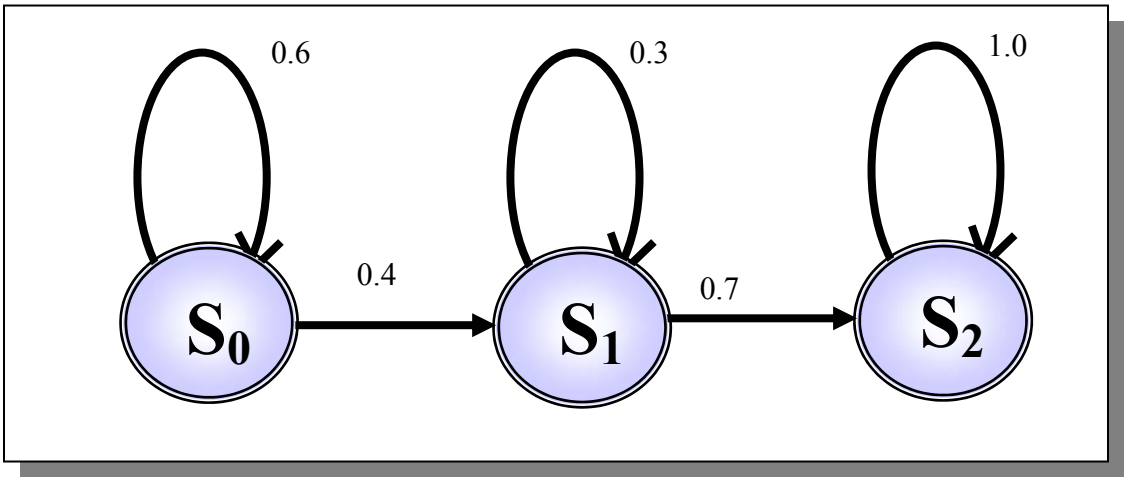
Problem 1. Assume a discrete left-to-right HMM shown below that can emit symbols "red", "green", "blue" with initial state distribution π, state transition matrix A and output distribution B_0, B_1, and B_2 for states 0, 1, and 2.



$$\pi = \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} \qquad A = \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.0 & 0.3 & 0.7 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} \text{red} & 0.80 \\ \text{green} & 0.15 \\ \text{blue} & 0.05 \end{bmatrix} \qquad B_1 = \begin{bmatrix} \text{red} & 0.65 \\ \text{green} & 0.10 \\ \text{blue} & 0.25 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \text{red} & 0.20 \\ \text{green} & 0.30 \\ \text{blue} & 0.50 \end{bmatrix}$$

1.1 Forward Algorithm

Using the **forward algorithm**, calculate the probability of observing the symbol sequence:

"**red red blue green red blue**"

Show all work!

1.2 Viterbi Algorithm

Using the **Viterbi algorithm**, determine the most likely state sequence that could have generated the symbol sequence:

"**red red blue green red blue**"

Again, show all work!

Problem 2: N-gram Language Models

You are given some training sentences to construct an n-gram language model:

```
<s> I WANT TO GO FROM DENVER TO BOSTON TOMORROW MORNING </s>
<s> I WOULD LIKE TO GO TO SEATTLE THIS AFTERNOON </s>
<s> DEPARTING SEATTLE </s>
<s> IN THE LATE MORNING </s>
<s> LATE MORNING GOING TO DENVER FROM BOSTON </s>
<s> FROM DENVER LEAVING IN THE MORNING ARRIVING IN BOSTON </s>
```

Assuming a bigram language model, compute the probability of generating this sentence:

```
<s> FROM DENVER TO SEATTLE  </s>
```

Show your work.

Problem 3: Token Passing Search Methodology

In your own words, briefly describe the methodology behind token passing and how tokens are used to represent search in large vocabulary speech recognition systems. Cover at minimum the following topics: (i) Maintenance of path scores, (ii) word-link records and their use in decoding the best word sequence, (iii) the fan-out problem of phonetic context modeling at cross-word positions, (iv) how language models are applied, and (v) token pruning methods. Use illustrations to provide insight if needed. Do not exceed 1 page of text!

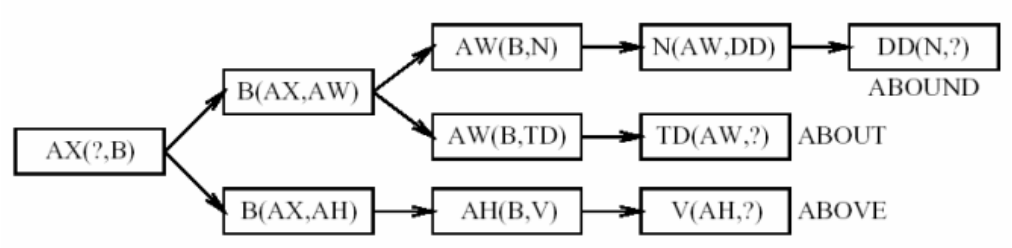
Problem 4: Lexical Prefix Tree

Given the following vocabulary and word pronunciations:

GO	G OW
GOING	G OW AX N
GOING(2)	G OW IX NG
TO	T AX
TO(2)	T IX
TO(3)	T UW
TODAY	T AX D EY
TODAY(2)	T UW D EY

```
TOMORROW                T AX M AA R OW
TOMORROW(2)             T UW M AA R OW
TUESDAY                 T UW Z D EY
TUESDAY(2)              T UW Z D IY
```

4.1 Draw a connected lexical prefix tree in terms of triphones that would be used to represent the search space. For example,



4.2 How many root nodes are needed to represent the search space?

4.3 How many total nodes are needed to represent the search space (assuming each triphone counts as one node)?

4.4 How many total nodes would have been needed to represent a similar linear (flat) search network constructed from triphones? What is the percent savings in terms of nodes? Please give details.