T-61.182 Special course in information science II
Information theory and machine learning

Exercise 1 (inference and variance)

For a normal distribution in two variables with

$$C = \begin{bmatrix} 1 & 0.75 \\ 0.75 & 0.75 \end{bmatrix}$$

as covariance and zero mean, compute the variance in terms of the first variable if the second one is observed, and vice versa.

Exercise 2 (Samples from a Gaussian Process prior)

Draw a sample $\boldsymbol{X}$ at random from the uniform distribution on $[0,1]^2$ and compute the corresponding covariance matrix $\boldsymbol{C}$. Use for instance the linear covariance function $C(\boldsymbol{x}, \boldsymbol{x}') = \boldsymbol{x}\boldsymbol{x}'$ and the Gaussian RBF covariance function $C(\boldsymbol{x}, \boldsymbol{x}') = \exp\left\{-\frac{1}{2\sigma^2}\|\boldsymbol{x} - \boldsymbol{x}'\|^2\right\}$.
Write a program which draws samples uniformly from the normal distribution $\mathcal{N}(0, \boldsymbol{C})$ (Hint: Compute the eigenvectors of $\boldsymbol{C}$ first). What difference do you observe when using different covariance functions?

Exercises are from Schölkopf & Smola: Learning with Kernels