

## T.61.5140 Machine Learning: Advanced Probabilistic Methods

Hollmén, Raiko (Spring 2008)

Problem session, 8th of February, 2008

<http://www.cis.hut.fi/Opinnot/T-61.5140/>

1. Consider a model for five binary variables  $x_i$ . What is the number of parameters needed to represent the distribution  $P(x_1, x_2, x_3, x_4, x_5)$  if no assumptions are made? How about if the model in Figure 1 is assumed? And how about, if additionally, the Markov assumption ( $P(x_{i+1} | x_i) = P(x_i | x_{i-1}), i = 2, 3, 4$ ) is made? Note: Sometimes only free parameters are counted, that is, for every sum of probabilities that equals 1, one parameter is determined by the others and is thus not counted. We can ignore this here and count all the parameters.

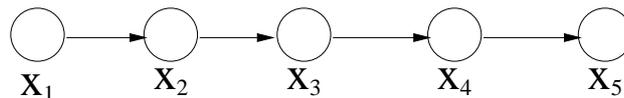


Figure 1: Problems 1 and 2.

2. For the Bayesian network in Figure 1, solve  $P(x_3 | x_1, x_5)$ .
3. Show that the property of there being no directed cycles in a directed graph follows from the statement that there exists an ordered numbering of the nodes such that for each node there are no links going to a lower-numbered node.
4. Using the d-separation criterion, show that the conditional distribution for a node  $x_i$  in a directed graph, conditioned on all of the nodes in the Markov blanket, is independent of the remaining variables in the graph.

Note: The next exercise session will be in two weeks (Feb 22).