T.61.5140 Machine Learning: Advanced Probablistic Methods
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1. Consider a bent coin and how to estimate the probability of tails μ. The random variable $X \in \{0, 1\}$ (heads=0, tails=1) is distributed accoring to the Bernoulli distribution with the parameter μ (see page 685 in Bishop, 2006).
(a) Derive a maximum likelihood estimator for μ and estimate $\hat{\mu}$ for the data set from the lecture (7 heads and 5 tails out of 12 tosses).
(b) Using a fair coin, what is the probability that out of 12 tosses, strictly more than 10 are heads (see Binomial distribution, page 686).

2. Compute the probability $P(C \mid X)$ of using each coin in the guessing game from the lecture (see Bayes' theorem, p. 15). There are two bent coins ($C \in \{c_1, c_2\}$) with different properties and the player guesses which coin was used after learning whether the toss was head or tails. The properties of the coins are: $P(X = t \mid C = c_1) = \theta_1$ and $P(X = t \mid C = c_2) = \theta_2$. The used coin is chosen randomly by $P(C = c_1) = \pi_1$ and $P(C = c_2) = \pi_2$ with $\pi_1 + \pi_2 = 1$.

3. The Naïve Bayes model has a class label C and observations $X_1, X_2, \ldots, X_6$ such that $P(X_1, X_2, X_3, X_4, X_5, X_6, C) = P(C)P(X_1|C)P(X_2|C)\ldots P(X_6|C)$.
(a) Simplify $P(X_1 \mid C, X_2)$
(b) Solve the classification problem: $P(C \mid X_1, X_2, \ldots, X_6)$

4. Draw a graphical representation of the models in problems 1, 2, and 3 where nodes represent random variables and arrows represent direct dependencies (see Bayesian Networks, page 360).