T-61.5100 Digital image processing, Exercise 9/07

Image compression / Image segmentation

1. The optimal quantizer is created by setting the reconstruction levels (codebook values) t_i so that the mean square error $\sum_{i=1}^{N} \int_{s_{i-1}}^{s_i} (s - t_i)^2 p(s) ds$ is minimized. (Here $s_0 = -\infty$, $s_N = \infty$, and $s_i = (t_i + t_{i-1})/2$, kun $i \neq 1$ ja $i \neq N$.) Where should the two reconstruction levels be placed, if the propability density function is

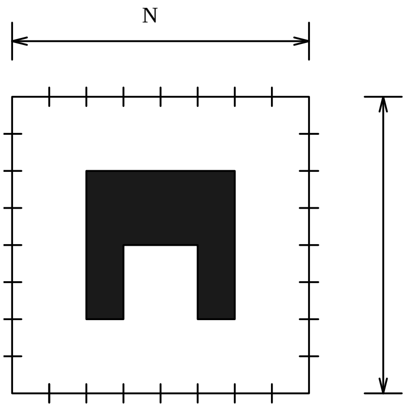
$$p(s) = \begin{cases} s+1, & \text{when } -1 < s < 0, \\ -s+1, & \text{when } 0 \leq s < 1, \\ 0 & \text{otherwise} \end{cases}$$

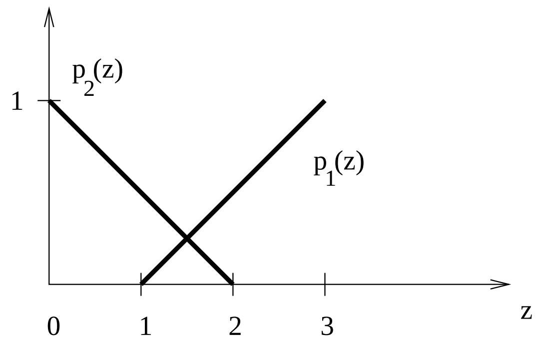
2. Inspect the encoder in Figure 8.28(a) in the textbook and ignore the symbol encoder part, because it causes no error. Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ be a subimage to be encoded. Transformed image $\boldsymbol{y}$ will be formed from $\boldsymbol{x}$ so that first some linear orthonormed transformation is applied (for example, Fourier transformation) $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{x}$. After this, $\boldsymbol{y}$ will be replaced by m first components of $\boldsymbol{y}'$ and the result is given to the quantizer. Let the average quadratic truncation error $e_m^2 = E\left\{\|\boldsymbol{y} - \boldsymbol{y}'\|^2\right\}$, the quantization error $e_q^2 = E\left\{\|\boldsymbol{v} - \boldsymbol{y}\|^2\right\}$, and the total error $e_T^2 = E\left\{\|\boldsymbol{v} - \boldsymbol{y}'\|^2\right\}$ be defined as where $\boldsymbol{v}$ is the output of the quantizer. Let us assume that truncation and quantization errors are uncorrelated and thus additive.

   (a) Show that the truncation error equals to the energy corresponding to the excluded components of $\boldsymbol{y}'$ $e_m^2 = \sum_{i=m+1}^{n} E\{y_i'^2\}$

   (b) Show that from the fact that truncation and quantization errors are uncorrelated follows $e_T^2 = e_m^2 + e_q^2$

3. Segment the image shown below (left) using the split and merge procedure. Let $P(R_1)$=TRUE if all pixels in R_1 have the same intensity. Show the quadtree corresponding to your segmentation.





4. Suppose that an image has the intensity distribution as shown above (right), where $p_1(z)$ corresponds to the intensity of the objects and $p_2(z)$ corresponds to the intensity of the background. Assuming that $P_1 = P_2$, find the optimal threshold between object and background pixels.

5. Assume that the image consists of small, non-overlapping bubbles, which have a mean grayscale value of $m_1 = 150$ and a variance $\sigma_1^2 = 400$. The background has mean $m_2 = 25$ and variance $\sigma_2^2 = 625$. The bubbles take up about 20 % of the image. Show a method based on thresholding which separates the bubbles from the background.