T-61.5100 Digital image processing, Exercise 7/07

Wavelets

1. Construct a fully populated approximation pyramid and corresponding prediction residual pyramid for the image

$$f(x,y) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}.$$

   Use 2×2 block neighborhood averaging for the approximation filter and omit the interpolation filter (see pp. 351 in the textbook).

2. Compute the Haar transform $\mathbf{T} = \mathbf{HFH}^{\mathrm{T}}$ of the 2×2 image

$$\mathbf{F} = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}.$$

   Compute also the inverse Haar transform $\mathbf{F} = \mathbf{H}^{\mathrm{T}}\mathbf{TH}$ of the obtained result.

3. Compute the two-dimensional wavelet transform with respect to Haar wavelets of the 2×2 image $\mathbf{F}$ in previous exercise. Draw the required filter bank and label all inputs and outputs with the proper arrays.

4. Draw wavelet $\psi_{3,3}(x)$ for the Haar wavelet function. Write an expression for $\psi_{3,3}(x)$ in terms of the Haar scaling function.

5. Compute the one-dimensional discrete wavelet transform (DWT) of function $f(0) = 1$, $f(1) = 4$, $f(2) = -3$, and $f(3) = 0$ with starting scale $j_0 = 1$. Then compute the inverse transform.