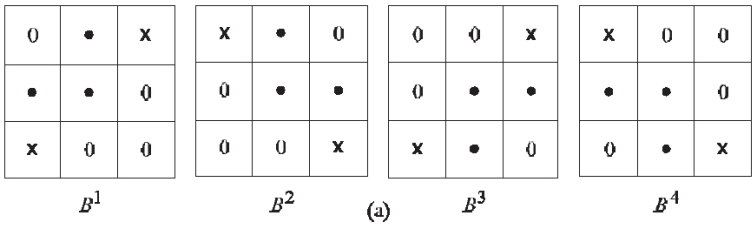
T-61.5100 Digital image processing, Exercise 6/07

1.

a) With reference to the discussion in Section 2.5.2, m-connectivity is used to avoid multiple paths that are inherent in 8-connectivity. In one-pixel-thick, fully connected boundaries, these multiple paths manifest themselves in the four basic patterns shown here:



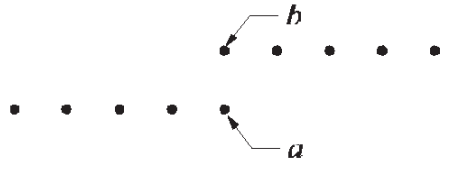
The solution to the problem is to use the hit-or-miss transform to detect the patterns and then to change the center pixel to 0, thus eliminating the multiple paths. A basic sequence of morphological steps to accomplish this is as follows:

$$
\begin{aligned}
X_1 &= A \circledast B^1 \\
Y_1 &= A \cap X_1^c \\
X_2 &= Y_1 \circledast B^2 \\
Y_2 &= Y_1 \cap X_2^c \\
X_3 &= Y_2 \circledast B^3 \\
Y_3 &= Y_2 \cap X_3^c \\
X_4 &= Y_3 \circledast B^4 \\
Y_4 &= Y_3 \cap X_4^c
\end{aligned}
$$

where A is the input image containing the boundary.

b) Only one pass is required. Application of the hit-or-miss transform using a given B^i finds all instances of occurrence of the pattern described by that structuring element.

c) The order does matter. For example, consider the sequence of points shown in next figure and assume that we are traveling from left to right. If B^1 is applied first, point a will be deleted and point b will remain after application of all other structuring elements. If, on the other hand, B^3 is applied first, point b will be deleted and point a will remain. Thus, we would end up with different (but of course, acceptable) m-paths.



2.

The morphological operation closing is used to close the outline of the zero in the image A given as

0	0	0	0	0	0	0	0	0
0	0	1	1	0	1	1	0	0
0	0	1	0	1	0	0	1	0
0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Closing is dilation followed by erosion. Dilation of A by B is the set of all points z such that $\hat{B}$ (B's reflection about its origin) and A overlap by at least one element,

$$A \oplus B = \{z|[(\hat{B})_z \cap A] \subseteq A\}.$$

Erosion of A by B is the set of all points z such that B is contained in A,

$$A \ominus B = \{z|(B)_z \subseteq A\}.$$

Closing is defined as

$$A \bullet B = (A \oplus B) \ominus B.$$

The structuring element chosen for B is

B's reflection about its origin, $\hat{B}$, is then

The result of dilation of the image A by B is

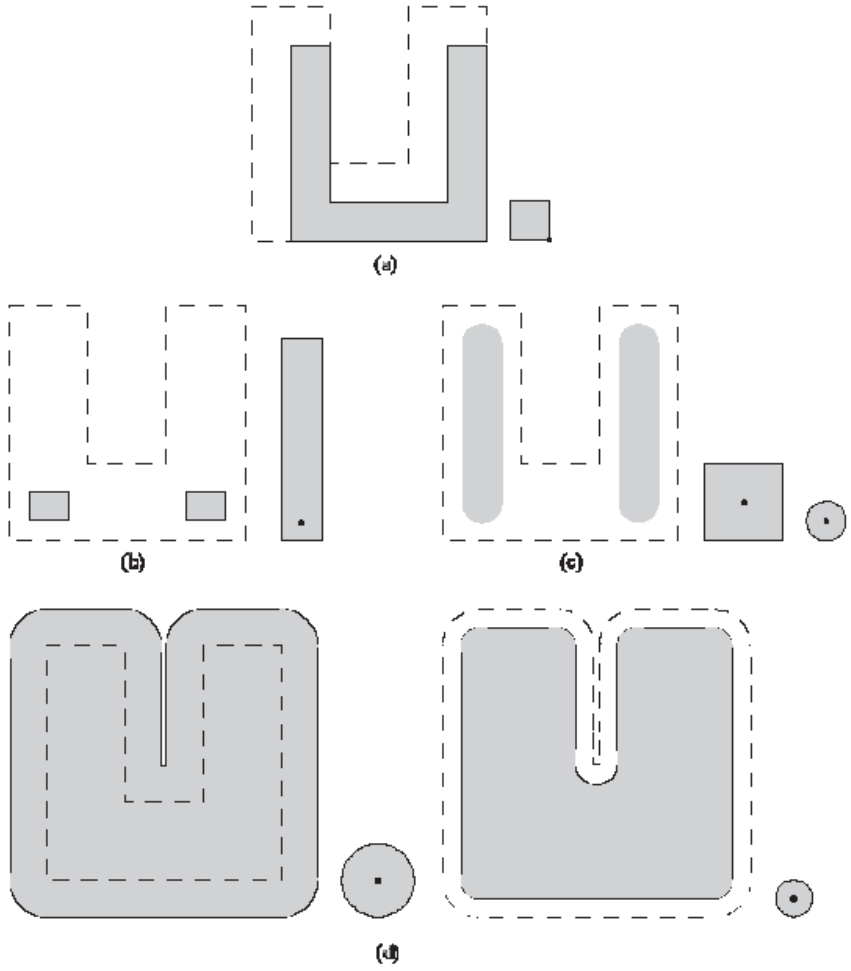
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0
0	0	1	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0

The previous image after erosion is

0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0
0	0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

3.

The center of each structuring element is shown as a black dot.
Solution (a) was obtained by eroding the original set (shown dashed) with the structuring element shown (note that the origin is at the bottom, right).
Solution (b) was obtained by eroding the original set with the tall rectangular structuring element shown.
Solution(c) was obtained by first eroding the image shown down to two vertical lines using the rectangular structuring element. This result was then dilated with the circular structuring element. **NOTE:** This solution is not entirely correct, after the initial eroding there is also a horizontal line connecting the two vertical ones.
Solution (d) was obtained by first dilating the original set with the large disk shown. Then dilated image was then eroded with a disk of half the diameter of the disk used for dilation.



4.

The solution is shown in the next figure. Although the images shown could be sketched by hand, they were done in MATLAB. The size of the original is 647 x 624 pixels. A disk structuring element of radius 11 was used. This structuring element was just large enough to encompass all noise elements, as given in the problem statement.
The images shown in the figure are: (a) erosion of the original, (b) dilation of the result, (c) another dilation, and finally (d) an erosion.
The first erosion (leftmost image) should take out all noise elements that do not touch the rectangle, should increase the size of the noise elements completely contained within the rectangle,

and should decrease the size of the rectangle. If worked by hand, the student may or may not realize that some imperfections are left along the boundary of the object. We do not consider this an important issue because it is scale-dependent, and nothing is said in the problem statement about this.
The first dilation (next image) should shrink the noise components that were increased in erosion, should increase the size of the rectangle, and should round the corners.
The next dilation should eliminate the internal noise components completely and further increase the size of the rectangle.
The final erosion (last image on the right) should then decrease the size of the rectangle. The rounded corners in the final answer are an important point that should be recognized by the student.