T-61.5100 Digital image processing, Exercise 5/07

Image restoration

1. Consider a linear, position invariant image degradation system with impulse response

$$h(x, y) = e^{-(x^2+y^2)}$$

Suppose the input to the system is an image consisting of a line of infinitesimal width located at $x = a$, and modeled by $f(x, y) = \delta(x - a)$. Assuming no noise, what is the output image $g(x, y)$?

2. During acquisition, an image undergoes uniform linear motion in the vertical direction for a time T_1. The direction of motion then switches to the horizontal direction for a time interval T_2. Assuming that the time it takes the image to change directions is negligible, and the shutter opening and closing times are negligible also, give and expression for the blurring function, $H(u, v)$.

3. Cannon [1974] suggested the power spectrum equalization filter $R(u, v)$ based on the premise of forcing the power spectrum of the restored image to equal the power spectrum of the original image:

$$S_{\hat{f}}(u, v) = |R(u, v)|^2 S_g(u, v) = S_f(u, v).$$

Find $|R(u, v)|$, the magnitude response of the restoration filter.

4. Find magnitude responses for the

   (a) inverse filter

   (b) power spectrum equalization filter

   (c) Wiener filter

   in the points (u, v) of frequency domain, where signal power spectrum $S_f(u, v)$, noise $S_n(u, v)$ power spectrum and magnitude response of point spread function (PSF) $H(u, v)$ have the following values:

$\lvert H(u,v)\rvert$	$S_f(u,v)$	$S_n(u,v)$	
0	0	N	• zero of PSF
0	S	0	
0	S	N	
H	0	N	• signal power zero
H	S	0	• no noise
1.0	3000.0	0.01	• close to uv-origin
0.7	0.7	0.01	• low frequencies
0.01	0.005	0.01	• high frequencies

5. (a) Show that the application of a 3×3-sized local mean mask can be replaced by 1×3 and 3×1 masks applied sequentially. Compare the amount of additions that are needed in both cases.

   (b) Compare the amounts of additions and multiplications that are needed in a general case, where a N×N mask is replaced by 1×N and N×1 masks and masks' coefficients are not equal to ones.

   (c) Depict the 3×3 Sobel gradient masks. Show for one of the Sobel masks that it can be separated as above into two one-dimensional masks.

   (d) Is it possible to separate the 3×3 discrete Laplace-operator?