

T-61.5100 Digital image processing, Exercise 3/07

Image enhancement in the frequency domain I

1. Show that the orthogonality condition

$$\sum_{x=0}^{N-1} e^{j2\pi rx/N} e^{-j2\pi ux/N} = \begin{cases} N, & \text{when } r - u = kN \\ 0, & \text{otherwise} \end{cases}$$

is correct, when r , u , N and k are integers.

2. Fourier transform the sequence $f(0) = 2, f(1) = 3, f(2) = 4, f(3) = 4$. Then calculate the inverse Fourier transform and compare the result with the original sequence.
3. Show that the Fourier transform of the convolution of two functions is the product of their Fourier transforms. For simplicity, assume 1-D functions.
4. A Gaussian lowpass filter in the frequency domain has the transfer function

$$H(u, v) = Ae^{-(u^2+v^2)/2\sigma^2}.$$

Show that the corresponding filter in the spatial domain has the form

$$h(x, y) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}.$$

5. Suppose that you form a lowpass spatial filter that averages the 4-neighbors of a point (x, y) , but excludes the point (x, y) itself.
 - (a) Find the equivalent filter $H(u, v)$ in the frequency domain.
 - (b) Show that $H(u, v)$ is a lowpass filter.
 - (c) Consider also the phase response $\phi(u, v) = \arg H(u, v)$.