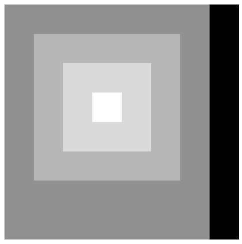
T-61.5100 Digital image processing, Exercise 2/07

Image enhancement in the spatial domain

1. Equalize the histogram of the 8×8 image below. The image has grey levels $0, 1, \ldots, 7$.



4	4	4	4	4	4	4	0
4	5	5	5	5	5	4	0
4	5	6	6	6	5	4	0
4	5	6	7	6	5	4	0
4	5	6	6	6	5	4	0
4	5	5	5	5	5	4	0
4	4	4	4	4	4	4	0
4	4	4	4	4	4	4	0

2. Assume that we have many noisy versions $g_i(x, y)$ of the same image $f(x, y)$, i.e.

$$g_i(x, y) = f(x, y) + \eta_i(x, y)$$

where the noise η_i is zero-mean and all point-pairs (x, y) are uncorrelated. Then we can reduce noise by taking the mean of all the noisy images

$$\bar{g}(x, y) = \frac{1}{M} \sum_{i=1}^{M} g_i(x, y).$$

Prove that

$$E\{\bar{g}(x, y)\} = f(x, y)$$

and

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{M} \sigma^2_{\eta(x,y)}$$

where $\sigma^2_{\eta(x,y)}$ is the variance of η and $\sigma^2_{\bar{g}(x,y)}$ the variance of $\bar{g}(x, y)$.

3. A two-variable continuous function's Laplace-operator is

$$\bigtriangledown^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

From this, it follows that

$$\mathcal{F}\{\bigtriangledown^2 f(x, y)\} = -(2\pi)^2 (u^2 + v^2) F(u, v),$$

where $F(u, v) = \mathcal{F}\{f(x, y)\}$. Determine the corresponding operator and the Fourier transform in the discrete case. Compare the result obtained to the continuous case.

4. Show that subtracting the Laplacian from an image is proportional to unsharp masking. Use the definition for the Laplacian in the discrete case.