T-61.5100 Digital image processing, Exercise 10/07

1.

a) Denote by c the given color and let its coordinates be denoted by (x_0, y_0). The distance between c and c_1 is

$$d(c, c_1) = [(x_0 - x_1)^2 + (y_0 - y_1)^2]^{1/2}.$$

Similarly the distance between c_1 and c_2 is

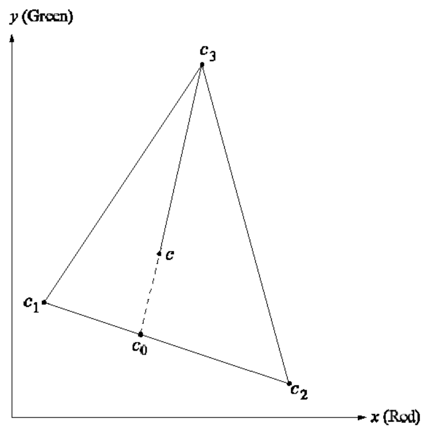
$$d(c_1, c_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2]^{1/2}.$$

The percentage p_1 of c_1 in c is

$$p_1 = \frac{d(c_1, c_2) - d(c, c_1)}{d(c_1, c_2)} \times 100.$$

The percentage p_2 of c_2 is simply $p_2 = 100 - p_1$. In the preceding equation we see, for example, that when $c = c_1$, then $d(c, c_1) = 0$ and it follows that $p_1 = 100\%$ and $p_2 = 0\%$. Similarly, when $d(c, c_1) = d(c_1, c_2)$, it follows that $p_1 = 0\%$ and $p_2 = 100\%$. Values in between are easily seen to follow from these simple relations.

b) Consider the following figure, in which c_1, c_2, and c_3 are the given vertices of the color triangle and c is an arbitrary color point contained within the triangle or on its boundary. The key to solving this problem is to realize that any color on the border of the triangle is made up of proportions from the two vertices defining the line segment that contains the point. The contribution to a point on the line by the color vertex opposite this line is 0%.



The line segment connecting points c_3 and c is shown extended (dashed segment) until it intersects the line segment connecting c_1 and c_2. The point of intersection is denoted c_0 . Because we have the values of c_1 and c_2, if we knew c_0, we could compute the percentages of c_1 and c_2 contained in c_0 by using the method in a). Denote the ratio of the content of c_1 and c_2 in c_0 by R_{12}. If we now add color c_3 to c_0, we know from a) that the point will start to move toward c_3 along the line shown. For any position of a point along this line we could determine the percentage of c_3 and c_0, again, by using the method described in a). The ratio R_{12} will remain the same for any point along the segment connecting c_3 and c_0.

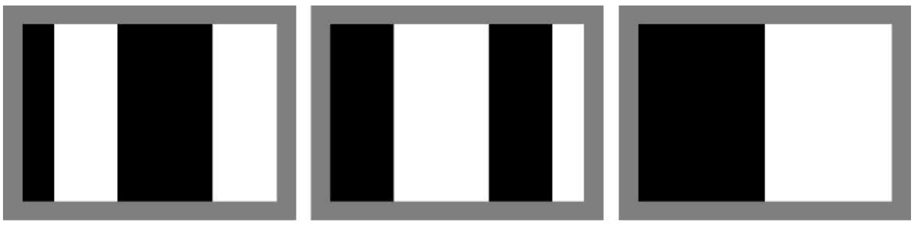
So, if we can obtain c_0, we can then determine the ratio R_{12}, and the percentage of c_3, in color c. The point c_0 is not difficult to obtain. Let $y = a_{12}x + b_{12}$ be the straight line containing points c_1 and c_2, and $y = a_{3c}x + b_{3c}$ the line containing c_3 and c. The intersection of these two lines gives the coordinates of c_0. The lines can be determined uniquely because we know the coordinates of the two point pairs needed to determine the line coefficients. Solving for the intersection in terms of these coordinates is straightforward, but tedious. Our interest here is

in the fundamental method, not the mechanics of manipulating simple equations so we do not give the details.
At this juncture we have the percentage of c_3 and the ratio between c_1 and c_2. Let the percentages of these three colors composing c be denoted by p_1, p_2, and p_3 respectively. Since we know that $p_1 + p_2 = 100 - p_3$, and that $p_1/p_2 = R_{12}$, we can solve for p_1 and p_2. Finally, note that this problem could have been solved the same way by intersecting one of the other two sides of the triangle. Going to another side would be necessary, for example, if the line we used in the preceding discussion had an infinite slope. Also a simple test to determine if the color of c is equal to any of the vertices should be the first step in the procedure, in this case no additional calculations would be required.

2.

a) For the given image, the maximum intensity and saturation requirement means that the RGB component values are 0 or 1. We can create the following table with 0 and 255 representing black and white, respectively.

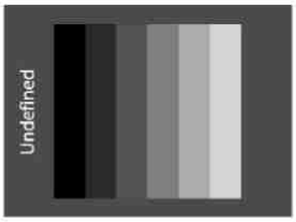
Color	R	G	B	Mono R	Mono G	Mono B
Black	0	0	0	0	0	0
Red	1	0	0	255	0	0
Yellow	1	1	0	255	255	0
Green	0	1	0	0	255	0
Cyan	0	1	1	0	255	255
Blue	0	0	1	0	0	255
Magenta	1	0	1	255	0	255
White	1	1	1	255	255	255
Gray	0.5	0.5	0.5	128	128	128



Red component Green component Blue component

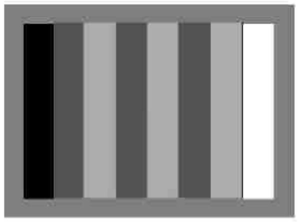
b) Using Eqs. (6.2-2) through (6.2-4), we get the results shown in the following:

Color	R	G	B	H	S	I	Mono H	Mono S	Mono I
Black	0	0	0	-	-	0	-	-	0
Red	1	0	0	0	1	0.33	0	255	85
Yellow	1	1	0	0.17	1	0.67	43	255	170
Green	0	1	0	0.33	1	0.33	85	255	85
Cyan	0	1	1	0.5	1	0.67	128	255	170
Blue	0	0	1	0.67	1	0.33	170	255	85
Magenta	1	0	1	0.83	1	0.67	213	255	170
White	1	1	1	-	0	1	-	0	255
Gray	0.5	0.5	0.5	-	0	0.5	-	0	128



Hue component

Saturation component



Intensity component

Note that, in accordance with Eq. (6.2-2), hue is undefined when $R = G = B$ since $\theta = \cos^{-1}(\frac{0}{0})$. In addition, saturation is undefined when $R = G = B = 0$ since Eq. (6.2-3) yields $S = 1 - \frac{3\min(0)}{3\cdot 0} = 1 - \frac{0}{0}$.

3.

Each component of the CMY image is a function of a single component of the corresponding RGB image,

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}.$$

In RGB color space, we know that

$$s_i = kr_i, \quad i = 1, 2, 3 \quad (\text{for } R, B, G).$$

And from the first equation, we know that the CMY components corresponding to the r_i and s_i are (for clarity, we will use a prime to denote the CMY components)

$$r_i^{'} = 1 - r_i \quad \text{and} \quad s_i^{'} = 1 - s_i.$$

Thus,

$$r_i = 1 - r_i^{'}$$

and

$$s_i^{'} = 1 - s_i = 1 - kr_i = 1 - k(1 - r_i^{'}) = kr_i^{'} + (1 - k).$$

4.

CIELab color coordinates for device-independent color representation:

$$\begin{aligned} \text{brightness:} \quad & L^* = 25(100Y/Y_0)^{1/3} - 16, \quad 1 \leq 100Y \leq 100 \\ \text{red-green content:} \quad & a^* = 500[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}], \\ \text{green-blue content:} \quad & b^* = 200[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}]. \end{aligned}$$

The values X, Y, and Z refer to intensities of red, green, and blue perceived by a subject. The set X_0, Y_0, and Z_0 is the tristimulus value of the reference white (e.g., zinc sulfate). (Note that the above definition is a bit different from the textbook's definition (p. 322). The condition under which the above equations are valid ($1 \leq 100Y \leq 100$) is the only difference.)

a) The new values of X, Y, and Z are $X_n = (1 + p)X = PX$, $Y_n = PY$, and $Z_n = PZ$. The change in L^* is given by

$$\Delta L^* = 25 \cdot 10^{2/3}[(PY/Y_0)^{1/3} - (Y/Y_0)^{1/3}] = 25 \cdot 10^{2/3}(P^{1/3} - 1)(Y/Y_0)^{1/3}.$$

The change in a^* is given by

$$\begin{aligned}\Delta a^* &= 500[(PX/X_0)^{1/3} - (PY/Y_0)^{1/3} - (X/X_0)^{1/3} + (Y/Y_0)^{1/3}]\\ &= 500(P^{1/3} - 1)[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}].\end{aligned}$$

The change in b^* is given by

$$\Delta b^* = 200(P^{1/3} - 1)[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}].$$

The change in s is determined by

$$(\Delta s)^2 = (\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2 = (P^{1/3} - 1)^2 A(X, Y, Z)$$

$$\Leftrightarrow |\Delta s| = |P^{1/3} - 1|\sqrt{A(X, Y, Z)}.$$

If values of X, Y, and Z are increased by 5 %, i.e., $P = 1.05$, the resulting change in s is

$$|\Delta s| = |1.05^{1/3} - 1|\sqrt{A(X, Y, Z)} = 0.0164\sqrt{A(X, Y, Z)}.$$

The relationship between the change in intensities of X, Y, and Z and in s is not linear.

b) For a 10 % change in X, Y, and Z:

$$|\Delta s| = |1.1^{1/3} - 1|\sqrt{A(X, Y, Z)} = 0.0323\sqrt{A(X, Y, Z)}.$$

c) The values of X_0, Y_0, and Z_0 are changed by 10 %. The changes in L^*, a^*, and b^* are

$$\begin{aligned}\Delta L^* &= 25 \cdot 10^{2/3}[(Y/PY_0)^{1/3} - (Y/Y_0)^{1/3}]\\ &= 25 \cdot 10^{2/3}(P^{-1/3} - 1)(Y/Y_0)^{1/3},\\ \Delta a^* &= 500(P^{-1/3} - 1)[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}],\\ \Delta b^* &= 200(P^{-1/3} - 1)[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}].\end{aligned}$$

The absolute change in s is

$$|\Delta s| = |P^{-1/3} - 1|\sqrt{A(X_0, Y_0, Z_0)}.$$

For a 10 % increase in X_0, Y_0, and Z_0

$$|\Delta s| = |1.1^{-1/3} - 1|\sqrt{A(X_0, Y_0, Z_0)} = 0.0313\sqrt{A(X_0, Y_0, Z_0)}.$$