

## Exercise 3, page 281

What are the impulse response and the frequency response (transfer function or MTF) of the 3x3 mean filter? Can you answer also for the median filter? Explain.

The **3x3 mean filter** has the impulse response:

$$h(m, n) = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \quad (1)$$

Its frequency response can be calculated using the 2D DFT:

$$F(k, l) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} h(m, n) \exp[-j2\pi \left( \frac{mk}{M} + \frac{nl}{N} \right)]. \quad (2)$$

$$F(0, 0) = \frac{1}{9} \sum_{m=0}^2 \sum_{n=0}^2 1 \cdot \exp[-j2\pi \left( \frac{m \cdot 0}{3} + \frac{n \cdot 0}{3} \right)] = 1. \quad (3)$$

$$F(1, 0) = \frac{1}{9} \sum_{m=0}^2 3 \exp[-j2\pi \frac{m}{3}] = 1 + \exp[-j\frac{2}{3}\pi] + \exp[-j\frac{4}{3}\pi] = 0. \quad (4)$$

$$F(k, l) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

The **median filter** is a nonlinear filter, so it cannot be calculated using convolution between an impulse response and the image. Hence, it does not have an impulse or a frequency response.