Image Coding and Data Compression

- Biomedical Images are of high spatial resolution and fine gray-scale quantisiation
  - Digital mammograms: 4,096x4,096 pixels with 12bit/pixel → 32MB per image
  - Volume data (CT & MRI): 512x512x64 with 16bit/pixel → 32MB per examination
- Health-care jurisdictions: store images ~ 7 years (or whole childhood)

Why digital images?

- Films deteriorate
- Easier accessible in a database
- Multiple copies without expenses
- Digital Image Analysis
- Allows for compression via coding

Why can we compress images?

- Redundancy
  - Code redundancy
  - Spatial redundancy (adjacent pixel intensities are correlated)
  - Psychovisual redundancy (this might not be an issue if one does digital image processing)
- Lossless vs. Lossy Compression
- “Diagnostically lossless”

Shannon's Source Coding Theorem

- The best achievable lossless compression is bounded by the Entropy.

  $H[x] <= l(x) < H[x] + 1$

- Necessary to coding sequences of symbols
- You can not do better except with lossy compression
- Entropy $H[x]$ is usually unknown.

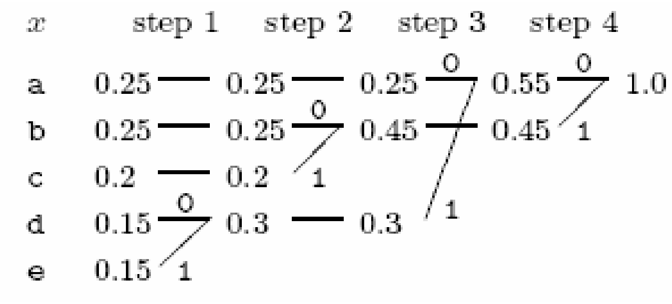
Fundamental technical terms

- *Alphabet*: set of symbols, e.g.: {0,1}.
- *Word*: finite sequence of symbols.
- *Code*: mapping of words form sources alphabet to code alphabet.
- *Uniquely decodable*: (bijective) code words are uniquely recognisable without separator (blanks).
- *Instantaneously decodable*: no codeword is prefix of an other.
- *Optimal*: if it is inst. decodable and has minimum code length given a sources pdf.

Huffman coding

- Lossless compression
- Optimal
- Requires pdf of gray level intensities
- → shortest code words to the most frequent *x*

Algorithm:



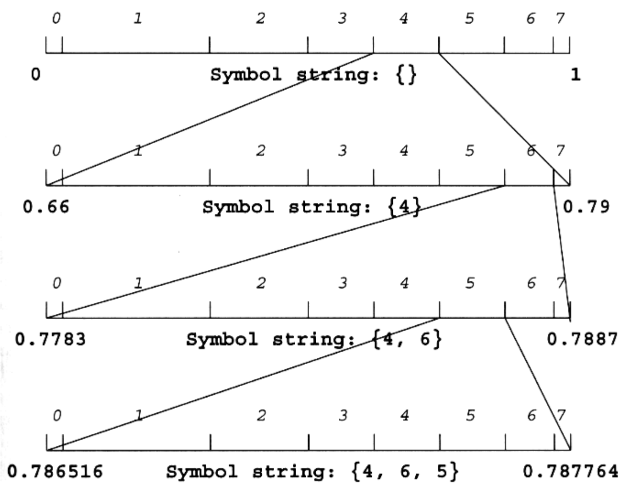
Result:

a_i	p_i	$h(p_i)$	l_i	$c(a_i)$
a	0.25	2.0	2	00
b	0.25	2.0	2	10
c	0.2	2.3	2	11
d	0.15	2.7	3	010
e	0.15	2.7	3	011

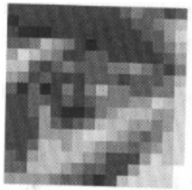
$E[l]$=2.3, $H(x)$=2.2855 bits

Arithmetic coding

- The source is represented as p_l and P_l
- Subdivide Intervals
- Does not use correlation between adjacent pixels



Example:



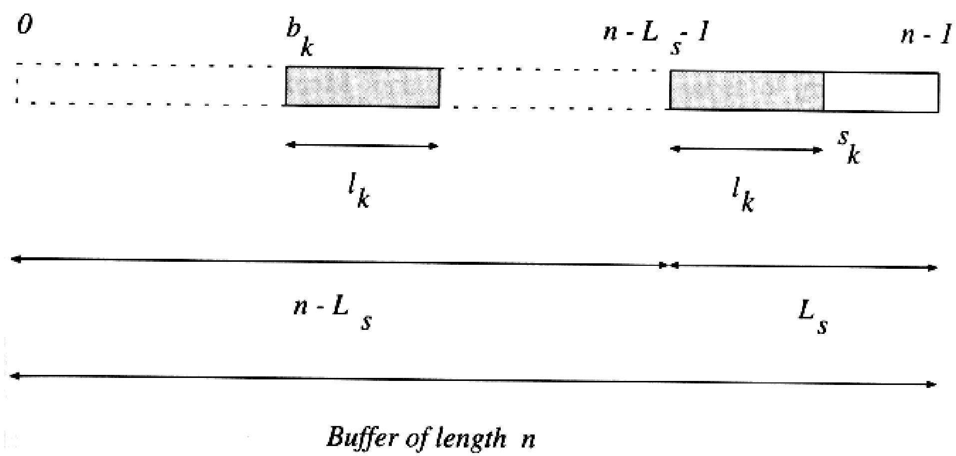
•Quantized 3 bit / pixel

Symbol l	Count	p_l	P_l	Interval
0	10	0.04	0.00	[0.00, 0.04)
1	77	0.30	0.04	[0.04, 0.34)
2	48	0.19	0.34	[0.34, 0.53)
3	33	0.13	0.53	[0.53, 0.66)
4	34	0.13	0.66	[0.66, 0.79)
5	32	0.12	0.79	[0.79, 0.91)
6	20	0.08	0.91	[0.91, 0.99)
7	2	0.01	0.99	[0.99, 1.00)

• Transmit binary code of integers

Lempel-Ziv Coding

- Not required to know the pdf
- Codebook code
- Based on look-up tables



Application: Source Coding of digitalised Mammograms

Image	Type	Size (pixels)	Entropy	Huffman	Arith.	LZW
1	Mammo.	$4,096 \times 1,990$	7.26	8.20	8.09	5.34
2	Mammo.	$4,096 \times 1,800$	7.61	8.59	8.50	5.76
3	Mammo.	$3,596 \times 1,632$	6.68	6.96	6.88	4.98
4	Mammo.	$3,580 \times 1,696$	7.21	7.80	7.71	4.68
5	Chest	$3,536 \times 3,184$	8.92	9.62	9.43	6.11
6	Chest	$3,904 \times 3,648$	9.43	9.83	9.81	6.27
7	Chest	$3,264 \times 3,616$	6.26	7.20	7.12	4.61
8	Chest	$4,096 \times 4,096$	8.65	9.39	9.35	5.83
9	Mammo.	$4,096 \times 2,304$	8.83	9.71	9.57	6.13
10	Chest	$4,096 \times 3,800$	8.57	9.42	9.33	5.99
	Average		7.94	8.67	8.58	5.57

Decorrelation

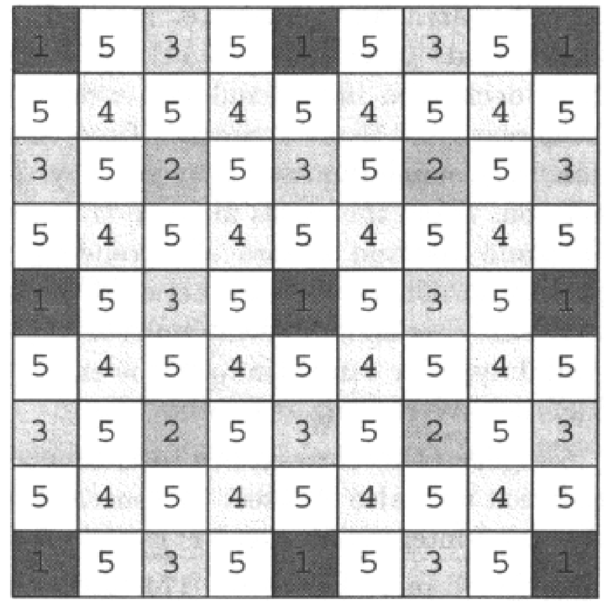
- Possible methods
  - Differentiation (remove commonality)
  - Transformation of bases
  - Model based Prediction
  - Interpolation
- The code mentioned before can be applied to decorrelated data.

Transform coding

- Orthogonal Transforms compress the energy of an image into a narrow region
  - Karhunen-Loeve Transform (PCA)
  - Discrete Cosine Transform (DCT)
  - Quantisation error in the coefficients of the transforms can lead to quantisation error in the reconstructed picture
  - The pdf of the transform coefficients tend to follow a Laplacian distribution

Interpolative Coding

- Code a sub-sampled image
- Derive the values of the remaining pixels via interpolation

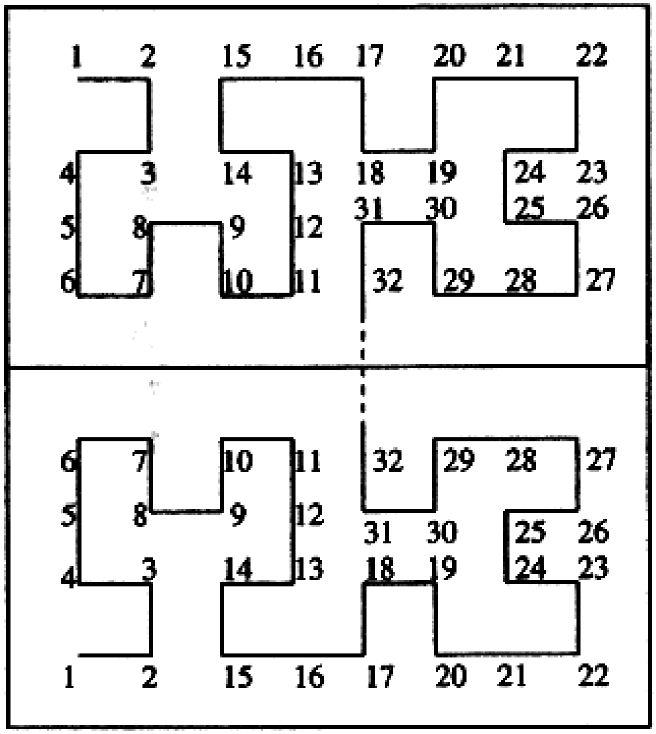


Predictive Coding

- Exploit spatial (or temporal) correlations
- Application to biomedical pictures promising
- Based on spatial linear autoregressive models
- Code initial conditions and the regression coefficients
- In case of lossless compression the prediction error needs to be transmitted

Penao-Scan (2d→1d)

- The curve fills the space continuously with out passing a point twice
- Original 2d locality is preserved compared to simple vectorisation
  - → e.g. use temporal AR models and code coefficients.



Region Growing based methods

- Segmentation based coding
- Compile an index-map
- Implementation difficulties: satisfactory segmentation → Region growing.
- Group spatially connected pixel

Application: Teleradiology

- Using digital transmission the resolution has do be sufficient so that subtle features can be reviled by radiologists (512 x 512 x 8bit), using 9,600 bps modems
- Targeting remote areas without radiologists.

Conclusion

- Coding and Compression are not the main topics of the book, since it focuses on image analysis
- Considering correlations and entropy relates to image analysis
- Software compression compete with cheaper media and greater bandwidth