T-61.5070 COMPUTER VISION, Exercise 6/08

1.

Grammars and languages are represented in Sec. 7.4.1, pp. 317–319. Methods for syntactic texture description are represented in Sec. 14.2., pp. 660–666. In the given grammar $G = [V_n, V_t, P, S]$, $V_n = \{x, y, z\}$ is the set of non-terminal symbols (variables), $V_t = \{A_1, C_1\}$ is the set of terminal symbols, and P is the set of substitution rules:

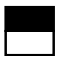
$$P: \quad \begin{array}{lllll} x & \rightarrow & \begin{array}{ccc} A_1 & - & y \\ | & & \\ x & & \end{array} & \text{or} & A_1 - y \\ & & & & \\ y & \rightarrow & C_1 - z & \text{or} & C_1 \\ & & & & \\ z & \rightarrow & A_1 - y & \text{or} & A_1 \end{array}$$

The starting symbol S, and the symbols A_1 and C_1 are not defined. Using x as the starting symbol produces the following chain:

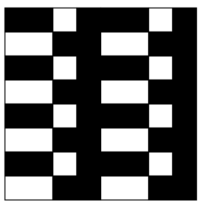
$$x \rightarrow \begin{array}{ccc} A_1 & - & y \\ | & & \\ x & & \end{array} \Rightarrow \begin{array}{ccccc} A_1 & - & C_1 & - & z \\ | & & & & \\ A_1 & - & y & & \\ | & & & & \\ x & & & & \end{array} \Rightarrow \begin{array}{ccccccc} A_1 & - & C_1 & - & A_1 & - & y \\ | & & & & & & \\ A_1 & - & C_1 & - & z & & \\ | & & & & & & \\ A_1 & - & y & & & & \end{array}$$

$$\Rightarrow \begin{array}{ccccccc} A_1 & - & C_1 & - & A_1 & - & C_1 \\ | & & & & & & \\ A_1 & - & C_1 & - & A_1 & - & y \\ | & & & & & & \\ A_1 & - & C_1 & - & z & & \end{array} \Rightarrow \begin{array}{ccccccc} A_1 & - & C_1 & - & A_1 & - & C_1 \\ | & & & & & & \\ A_1 & - & C_1 & - & A_1 & - & C_1 \\ | & & & & & & \\ A_1 & - & C_1 & - & A_1 & - & C_1 \end{array}$$

Therefore, if the symbols A_1 and C_1 are defined as

$A_1 =$  and $C_1 =$ 

then the grammar generates the following surface pattern:



2.

Co-occurrence matrices

The use of co-occurrence matrices in texture description is described in Sec. 14.1.2, pp. 651–653. The formalism of co-occurrence matrices represented below is somewhat different from that in the textbook. Assume an image function $f(\boldsymbol{x})$ in which $\boldsymbol{x} = [x\ y]^T$, $x \in \{1, 2, \ldots, X\}$ and $y \in \{1, 2, \ldots, Y\}$. X and Y are the sizes of the image in x- and y-directions, respectively. The image is digitized using G gray levels $g = 0, 1, \ldots, G-1$. Let $\boldsymbol{d} = [\Delta x\ \Delta y]^T$ be a displacement vector, in which $0 \leq \Delta x < X$ and $-Y < \Delta y < Y$. The co-occurrence matrix $P_d(i, j)$ is defined as a matrix, the (i, j)th element of which is the number of appearances of gray level values i and j with separation and direction determined by the displacement $\boldsymbol{d}$

$$P_d(i, j) = \#\{\boldsymbol{x} \mid f(\boldsymbol{x}) = i, f(\boldsymbol{x} + \boldsymbol{d}) = j\},$$

where # represents the number of elements in the set. The matrix is normalized by dividing each entry with $R = \sum_{i,j} P_d(i, j)$ which is the number of pixel pairs used in computing the matrix. The normalized matrix is $p_d(i, j) = P_d(i, j)/R$.
When the texture is coarse and the displacement $\boldsymbol{d}$ is short in comparison with the texture elements, the examined pixel pairs $\boldsymbol{x}$ and $\boldsymbol{x} + \boldsymbol{d}$ have similar values. Therefore, the high values of $p_d(i, j)$ are concentrated on the main diagonal. For a fine texture, the values of $p_d(i, j)$ are spread out quite uniformly. If a texture is directional then the co-occurrence matrices obtained for displacements along the texture direction are characterized by high values along the main diagonal. Displacements in other directions produce co-occurrence matrices with scattered values.

Exercise

Define the two displacements as $\boldsymbol{d_{10}} = [1\ 0]^T$ and $\boldsymbol{d_{01}} = [0\ 1]^T$. The first image and its two unnormalized co-occurrence matrices $P_{10}(i, j)$ and $P_{01}(i, j)$ are

2	0	0
1	2	0
0	2	1
0	0	2

P_{10}	0	1	2
0	2	0	2
1	0	0	1
2	2	1	0

P_{01}	0	1	2
0	2	1	1
1	1	0	1
2	1	1	1

.

The number of pixel pairs, R, used in computing the matrix is 8 for the displacement $(1, 0)$ and 9 for the displacement $(0, 1)$. The corresponding contrast and entropy measures are

$$C = \sum_i \sum_j (i - j)^2 P_d(i, j)/R$$

$$\begin{aligned} C_{10} =& [(0-2)^2 \cdot 2 + (1-2)^2 + (2-0)^2 \cdot 2 + (2-1)^2]/8 = 18/8 = 2.25 \\ C_{01} =& [(0-1)^2 + (0-2)^2 + (1-0)^2 + (1-2)^2 + (2-0)^2 + (2-1)^2]/9 = 12/9 \approx 1.33 \end{aligned}$$

$$E = -\sum_i \sum_j \frac{P_d(i, j)}{R} \log \frac{P_d(i, j)}{R}$$

$$\begin{aligned} E_{10} &= -\left(3 \cdot \frac{2}{8} \log \frac{2}{8} + 2 \cdot \frac{1}{8} \log \frac{1}{8} + 4 \cdot 0 \log 0\right) = -\frac{6}{8} \log 2 + \log 8 \approx 0.68 \\ E_{01} &= -\left(1 \cdot \frac{2}{9} \log \frac{2}{9} + 7 \cdot \frac{1}{9} \log \frac{1}{9} + 1 \cdot 0 \log 0\right) = -\frac{2}{9} \log 2 + \log 9 \approx 0.89 \end{aligned}$$

Notice, that we have chosen to use $\log_{10}$, while the book uses $\log_2$. We have also defined $0 \log 0 = 0$, since $\lim_{x \to 0} x \log x = 0$.
The second image and its two unnormalized co-occurrence matrices are

0	2	0
1	0	2
0	2	0
1	0	2

P_{10}	0	1	2
0	0	0	4
1	2	0	0
2	2	0	0

P_{01}	0	1	2
0	0	2	3
1	1	0	0
2	3	0	0

.

The corresponding contrast and entropy measures are

$$
\begin{aligned}
C_{10} =&[(0-2)^2 \cdot 4 + (1-0)^2 \cdot 2 + (2-0)^2 \cdot 2]/8 = 26/8 = 3.25 \\
C_{01} =&[(0-1)^2 \cdot 2 + (0-2)^2 \cdot 3 + (1-0)^2 + (2-0)^2 \cdot 3]/9 = 27/9 = 3.00 \\
E_{10} =& -(\frac{4}{8}\log\frac{4}{8} + 2\cdot\frac{2}{8}\log\frac{2}{8}) = -\frac{12}{8}\log 2 + \log 8 \approx 0.45 \\
E_{01} =& -(2\cdot\frac{3}{9}\log\frac{3}{9} + \frac{2}{9}\log\frac{2}{9} + \frac{1}{9}\log\frac{1}{9}) = -\frac{6}{9}\log 3 + \frac{2}{9}\log 2 + \log 9 \approx 0.57
\end{aligned}
$$

3.

Laws masks are derived from three simple vectors:

$$
\begin{aligned}
L_3 &= \begin{bmatrix}1 & 2 & 1\end{bmatrix} && \text{(center-weighted local average)} \\
E_3 &= \begin{bmatrix}-1 & 0 & 1\end{bmatrix} && \text{(edge detection)} \\
S_3 &= \begin{bmatrix}-1 & 2 & -1\end{bmatrix} && \text{(spot detection)}
\end{aligned}
$$

The following nine masks are obtained by multiplying these vectors with themselves or each other:

$$
L_3^T \times L_3 = \begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix} \quad
L_3^T \times E_3 = \begin{bmatrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 0 & 1\end{bmatrix} \quad
L_3^T \times S_3 = \begin{bmatrix}-1 & 2 & -1\\ -2 & 4 & -2\\ -1 & 2 & -1\end{bmatrix}
$$

$$
E_3^T \times L_3 = \begin{bmatrix}-1 & -2 & -1\\ 0 & 0 & 0\\ 1 & 2 & 1\end{bmatrix} \quad
E_3^T \times E_3 = \begin{bmatrix}1 & 0 & -1\\ 0 & 0 & 0\\ -1 & 0 & 1\end{bmatrix} \quad
E_3^T \times S_3 = \begin{bmatrix}1 & -2 & 1\\ 0 & 0 & 0\\ -1 & 2 & -1\end{bmatrix}
$$

$$
S_3^T \times L_3 = \begin{bmatrix}-1 & -2 & -1\\ 2 & 4 & 2\\ -1 & -2 & -1\end{bmatrix} \quad
S_3^T \times E_3 = \begin{bmatrix}1 & 0 & -1\\ -2 & 0 & 2\\ 1 & 0 & -1\end{bmatrix} \quad
S_3^T \times S_3 = \begin{bmatrix}1 & -2 & 1\\ -2 & 4 & -2\\ 1 & -2 & 1\end{bmatrix}
$$

These masks can be shown to span the space of 3×3 neighborhoods, i.e., any 3×3 array is a linear combination of them.

Properties of 3×3 Laws masks:

- L_3L_3 ($L_3^T \times L_3$) calculates the center-weighted average.
- L_3E_3 and E_3L_3 are classical (center-weighted) edge detectors that detect vertical and horizontal edges. They are similar to Sobel masks.
- L_3S_3 and S_3L_3 are (center-weighted) line detectors that will detect vertical and horizontal lines. E_3E_3 and S_3S_3 are also line detectors that will detect lines regardless of their orientation. They are similar to Frei-Chen line detector masks T_6 and T_7.
- E_3S_3 and S_3E_3 are included just for completeness, they don't detect any specific structures.

4.

The Fourier transform of region R in image $f(x,y)$ is defined by

$$F(u,v) = \iint_R e^{-j2\pi(ux+vy)} f(x,y)\,dx\,dy$$

The real valued Fourier power spectrum, denoted $|F(u,v)|^2$ is defined by

$$|F(u,v)|^2 = F(u,v)F^*(u,v).$$

The expression of Fourier power spectrum in polar coordinates $|F(r,\theta)|^2$ where $r = \sqrt{u^2+v^2}$ and $\theta = \arctan(v/u)$ provides measures for texture coarseness and directionality (see Sec. 14.1.1, p. 650).
The Fourier power spectrum of a coarse texture has largest values near the origin. The finer texture, the further the spectral peaks are from the origin. Therefore, the texture coarseness may be analyzed by examining rings $\phi_r(r)$ with different radii r, or averages at regular intervals $r_1 \leq r \leq r_2$

$$\phi_r(r) = \int_0^{\pi} |F(r,\theta)|^2\,d\theta.$$

Directionality, lines or edges in direction θ, results in high spectral values along the perpendicular direction $\theta + \frac{\pi}{2}$. Directionality may be analyzed by examining wedge-shaped regions $\theta_1 \leq \theta \leq \theta_2$ in

$$\phi_\theta(\theta) = \int_0^{\infty} |F(r,\theta)|^2\,dr.$$