## T-61.5070 COMPUTER VISION, Exercise 1/08

### 1.

The hexagonal grid is depicted in Fig. 1. The coordinates give the memory locations of grid elements. The six nearest neighbors of element $(2,1)$ in Fig. 1 are

$$N_{(2,1)} \in \{(3,0),(2,0),(1,0),(1,1),(2,2),(3,1)\}.$$

The nearest neighbors of each grid element are

$$N_{(i,j)} = \begin{cases} (i+1,j-1),(i,j-1),(i-1,j-1),(i-1,j),(i,j+1),(i+1,j) \\ \quad \text{if } i \text{ is even,} \\ (i+1,j),(i,j-1),(i-1,j),(i-1,j+1),(i,j+1),(i+1,j+1) \\ \quad \text{if } i \text{ is odd.} \end{cases} \tag{1}$$

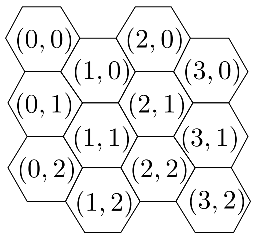


Figure 1: Hexagonal grid in memory.

The average for six nearest neighbors is calculated and substituted to each element, the image boundaries are not considered. This can be done by the following program:

```
for(i=1;i<N-1;i=i+2) /* process odd-i elements */
  for(j=1;j<N-1;j=j+1)
    ave[i][j] = (x[i][j] + x[i+1][j] + x[i][j-1] +
                 x[i-1][j] + x[i-1][j+1] + x[i][j+1] +
                 x[i+1][j+1])/7;

for(i=2;i<N-1;i=i+2) /* process even-i elements */
  for(j=1;j<N-1;j=j+1)
    ave[i][j] = (x[i][j] + x[i+1][j-1] + x[i][j-1] +
                 x[i-1][j-1] + x[i-1][j] + x[i][j+1] +
                 x[i+1][j])/7;
```

Fig. 2 depicts distances in a hexagonal grid. Euclidian distances are calculated in a hexagonal grid from displacements $d_i$ and $d_j$ in directions $i$ and $j$.

$$d(\boldsymbol{p_1},\boldsymbol{p_2}) = \sqrt{d_i^2 + d_j^2}. \tag{2}$$

If $i_1 - i_2$ is even, i.e. both elements are either in $C_e \in \{c_0, c_2, c_4, \cdots\}$ or in $C_o \in \{c_1, c_3, c_5, \cdots\}$, the displacement $d_i$ is

$$d_i = \frac{\sqrt{3}(i_1 - i_2)}{2} \tag{3}$$

and the displacement $d_j$ is simply

$$d_j = j_1 - j_2. \tag{4}$$

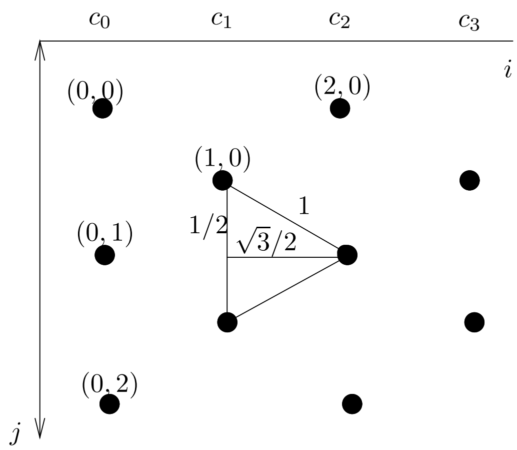


Figure 2: Distances in a hexagonal grid.

When $i_1 - i_2$ is odd, the displacement $d_i$ is obtained from Eq. 3 but displacement $d_j$ from Eq. 4 would be 0.5 units too large for even $i_1$ ($i_1 \in C_e$) and 0.5 units too small for an odd $i_1$ ($i_1 \in C_o$). Therefore, $d_j$ for an even $i_1$ is

$$d_j = j_1 - j_2 - \frac{1}{2}. \tag{5}$$

and for an odd $i_1$

$$d_j = j_1 - j_2 + \frac{1}{2}. \tag{6}$$

Euclidean distances may be computed with the following program

```
diff = j1 - j2;
if (((i1 - i2) % 2) != 0) {
  if ((i1%2) == 0)
    diff -= 0.5;
  else
    diff += 0.5;
}
dist = diff * diff;
diff = i1 - i2;
dist += 0.75 * diff * diff;
dist = sqrt(dist);
```

## 2.

Position-dependent brightness correction is represented in Sec. 4.1, pp. 58–59. The correction is made, for each image pixel $(i)$, with the light and dark calibration targets, $g_l(i)$ and $g_d(i)$. For a linear degradation, the degraded image, $f(i)$, is given by

$$f(i) = k(i)g(i) + b(i) \tag{7}$$

The calibration values for $k(i)$ and $b(i)$ of each pixel are obtained from the equation pair

$$\begin{cases} f_l(i) = k(i)g_l(i) + b(i) \\ f_d(i) = k(i)g_d(i) + b(i) \end{cases}. \tag{8}$$

The systematic brightness error can be suppressed by

$$g(i) = \frac{f(i) - b(i)}{k(i)}. \tag{9}$$

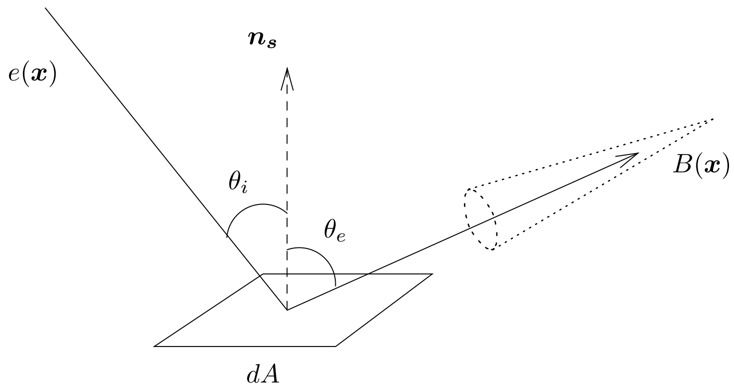


Figure 3: Radiometric image formation.

### 3.

The radiometric image formation is depicted in Fig. 3. The following definitions are made:

$e(\boldsymbol{x})$ Incident surface irradiance, which is the power per unit area of radiant energy falling on a surface. It is measured in units of watts per square meter ($W/m^2$).

$B(\boldsymbol{x})$ Surface radiance, which is the power per unit foreshortened area emitted into a unit solid angle. It is measured in units of watts per square meter per steradian ($W/m^2/sr$). Radiance can be a function of the viewing angle and the spectral wavelength and bandwidth.

$r(\boldsymbol{x})$ Surface reflectivity function (a.k.a. the reflectance, the reflection coefficient, or the bidirectional reflectance distribution function), which is the ratio of the radiant power per unit area per steradian reflected by the surface to the radiant power per unit area incident on the surface. It can be a function of the incident angle of the radiance, the viewing angle of the sensor, and the spectral wavelength and bandwidth.

$\theta_i$ Incidence angle.

$\theta_e$ Emittance angle.

The incident surface irradiance $e(\boldsymbol{x})$ is captured by an infinitesimal surface element $dA$. The power is given by the projection $e(\boldsymbol{x}) \cdot dA \cos\theta_i$. The amount of surface illumination seen by a viewer is given by $B(\boldsymbol{x}) \cdot dA \cos\theta_e$. The surface reflectivity function relates these two quantities by the ratio

$$r(\boldsymbol{x}) = \frac{B(\boldsymbol{x}) \cos\theta_e}{e(\boldsymbol{x}) \cos\theta_i} \tag{10}$$

In this case we want to calculate the surface radiance as seen from a certain angle $\theta_e$, i.e. the foreshortened $B(\boldsymbol{x}) \cdot \cos\theta_e$. Lets call this simply $B_e(\boldsymbol{x})$. Then we get:

$$B_e(\boldsymbol{x}) = e(\boldsymbol{x}) \cos\theta_i \; r(\boldsymbol{x}). \tag{11}$$

Lambertian surfaces have the property that under uniform illumination they look equally bright from any direction. The amount of light reflected from a unit area goes down as the cosine of the viewing angle, but the amount of area seen in any solid angle goes up as the reciprocal of the cosine of the viewing angle. Because we relate the perceived brightness to radiant power per solid angle, Lambertian surfaces seem to have the same brightness independent of viewing angle. This leads to the surface reflectivity function being constant for Lambertian surfaces, i.e. $r(\boldsymbol{x}) = r_0$. This property is also sometimes given as the definition of a Lambertian surface.

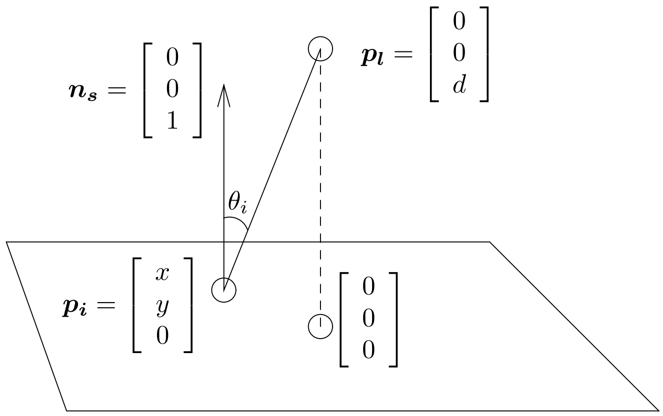


Figure 4: A Lambertian plane illuminated by a point source of light.

**a)** A Lambertian plane is illuminated by a point source of light at $\boldsymbol{p_l}$, Fig. 4. The reflectivity function $r(\boldsymbol{p_i})$ of the plane is constant, $r(\boldsymbol{p_i}) = r_0$. The irradiance $e(\boldsymbol{p_i})$ varies inversely as the square of the distance from the illuminated surface to the source (law of inverse squares), $e(\boldsymbol{p_i}) = I_0/d_s^2$.

$$B_e(\boldsymbol{p_i}) = e(\boldsymbol{p_i}) \cos\theta_i r(\boldsymbol{p_i}) = \frac{I_0 \cos\theta_i r_0}{d_s^2} = \frac{I_0 r_0}{\|\boldsymbol{p_l} - \boldsymbol{p_i}\|^2} \frac{\boldsymbol{n_s} \cdot (\boldsymbol{p_l} - \boldsymbol{p_i})}{\|\boldsymbol{n_s}\| \|\boldsymbol{p_l} - \boldsymbol{p_i}\|}$$

$$= I_0 r_0 \frac{[0\ 0\ 1][-x\ \ -y\ d]^T}{(x^2 + y^2 + d^2)^{3/2}} = \frac{I_0 r_0 d}{(x^2 + y^2 + d^2)^{3/2}} \tag{12}$$

The brightest point on the plane is illuminated from normal direction ($x = y = 0 \Rightarrow B_e(\boldsymbol{p_i}) = I_0 r_0 / d^2$) and around this point the brightness decreases concentrically with decreasing illumination angle.

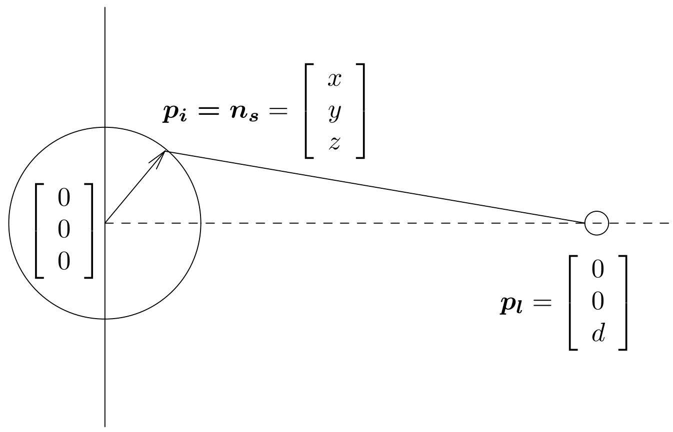


Figure 5: A Lambertian sphere illuminated by a point source of light.

**b)** A Lambertian sphere is illuminated by a point source of light $\boldsymbol{p_l}$, Fig. 5. Assume that the sphere has a unit radius, i.e., $\sqrt{x^2 + y^2 + z^2} = 1$.

$$B_e(\boldsymbol{p_i}) = I_0 r_0 \frac{[x\ y\ z][-x\ \ -y\ d - z]^T}{(x^2 + y^2 + z^2 + d^2 - 2dz)^{3/2}}$$

$$= I_0 r_0 \frac{-x^2 - y^2 - z^2 + dz}{(d^2 - 2dz + 1)^{3/2}} = \frac{I_0 r_0 (dz - 1)}{(d^2 - 2dz + 1)^{3/2}} \tag{13}$$

The brightest point is illuminated from normal direction ($z = 1 \Rightarrow B_e(\boldsymbol{p_i}) = I_0 r_0/(d-1)^2$). The brightness reaches zero at $z = 1/d$ where the light hits the sphere tangentially.

**4.**

Two point sources of light illuminate a Lambertian plane, Fig. 6. The axis has been selected so that the lights are at points $\boldsymbol{p_l^A} = [-e\ 0\ d]^T$ and $\boldsymbol{p_l^B} = [e\ 0\ d]^T$. The irradiance is $e(\boldsymbol{p_i}) = I_0/d_s^2$. The reflectivity function $r(\boldsymbol{p_i}) = r_0$ is constant. The surface radiance at point $\boldsymbol{p_i} = [x\ y\ 0]^T$ is then

$$B_e(\boldsymbol{p_i}) = I_0 r_0 \left(\frac{\cos\theta_i^A}{d_A^2} + \frac{\cos\theta_i^B}{d_B^2}\right) = I_0 r_0 \left(\frac{\boldsymbol{n_s} \cdot (\boldsymbol{p_l^A} - \boldsymbol{p_i})}{\|\boldsymbol{n_s}\|\|\boldsymbol{p_l^A} - \boldsymbol{p_i}\|^3} + \frac{\boldsymbol{n_s} \cdot (\boldsymbol{p_l^B} - \boldsymbol{p_i})}{\|\boldsymbol{n_s}\|\|\boldsymbol{p_l^B} - \boldsymbol{p_i}\|^3}\right)$$

$$= I_0 r_0 \left(\frac{[0\ 0\ 1][(-e-x)\ (-y)\ d]^T}{((e+x)^2 + y^2 + d^2)^{3/2}} + \frac{[0\ 0\ 1][(e-x)\ (-y)\ d]^T}{((e-x)^2 + y^2 + d^2)^{3/2}}\right)$$

$$= I_0 r_0 \left(\frac{d}{((e+x)^2 + y^2 + d^2)^{3/2}} + \frac{d}{((e-x)^2 + y^2 + d^2)^{3/2}}\right) \tag{14}$$

The radiance $B_e(\boldsymbol{p_i})$ is a sum of two radiances decreasing concentrically around the brightest point. The plane has two radiance maxima, at $\boldsymbol{p_i} = [-e\ 0\ 0]^T$ and $\boldsymbol{p_i} = [e\ 0\ 0]^T$. The isoradiance curves on the plane are depicted in Fig. 7 ($e = 1$).

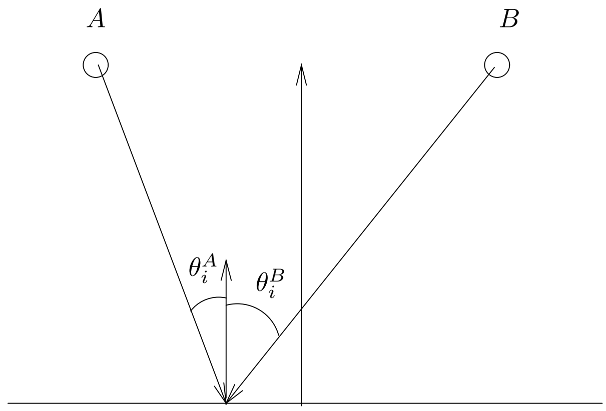


Figure 6: A Lambertian plane illuminated by two point sources of light.

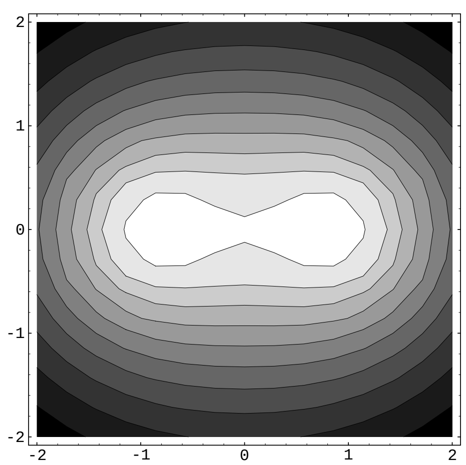


Figure 7: The variation of luminance at illumination by two point sources.