T-61.5070 COMPUTER VISION, Exercise 11/08

1.

Focus of expansion (FOE, *F. laajentumispiste*) is described in Sec. 15.2.4, pp. 693–696. For an object moving with constant velocity, all image plane flow vectors intersect at FOE. A change in motion direction results in change of velocity, and a change in FOE location. If the movement of object points cannot be described by simple constant velocity translation, e.g. when the object is rotating, no FOE may be determined. In general, a FOE does not exist for objects moving with nonconstant 3-D velocities. For the special case when the object speed varies while the the velocity direction stays constant the FOE can be determined, however.

2.

A moving rectangle has been chosen as an example of image motion for this exercise, Fig. 1.

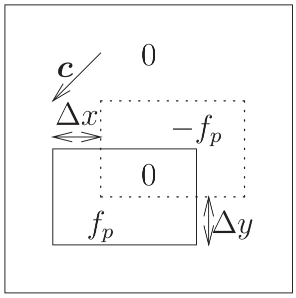

Figure 1: Ternary image example.

Gray-scale values are f_p for the object and 0 for the background. New images of the moving rectangle are taken at a constant interval T. The difference image between two subsequent frames has four regions with three intensities:

1. A region of intensity $-f_p$, because of the uncovering of the background over the time interval;
2. A region of intensity f_p, because of covering the background over the time interval;
3. A region between the above two of intensity 0, because of the presence of the object in this region in both images; and
4. A region of intensity 0, because of the unchanged background in both images.

The difference image thus has three different possible values: $-f_p$, 0 and f_p, hence the name ternary image.

a) Determine changes Δx and Δy in the direction from the region with intensity of $-f_p$ to the region with intensity f_p. The velocity is $\boldsymbol{c} = [\frac{\Delta x}{T}\ \frac{\Delta y}{T}]^{\mathrm{T}}$.

b) The background intensity is thresholded to value f_0 instead of 0. If

$$f_0 > f_p \Rightarrow f_p - f_0 < 0,$$

then the direction of velocity is from the region with positive intensity values to the region with negative values. If

$$f_0 < f_p \Rightarrow f_p - f_0 > 0,$$

then the situation is the same as in section **a)**.

c) If the background intensity f_0(x) varies being sometimes identical to, or higher or lower than the object intensity, then the differentiation between object and background becomes difficult. In that case a proper segmentation cannot be obtained with simple thresholding; its use in this case may lead to an inaccurate and erroneous local velocity.

3.

Computation of optical flow is represented in Sec. 15.2.1, pp. 686–689. Algorithm 15.1, p. 687, determines optical flow from a pair of consecutive images $f(x, y, t)$ and $f(x, y, t{+}1)$. The velocity components $u(x, y)$ and $v(x, y)$ in the x and y directions, respectively, are computed iteratively according to the equations:

$$u^k = \bar{u}^{k-1} - f_x \frac{P}{D}$$

$$v^k = \bar{v}^{k-1} - f_y \frac{P}{D}, \tag{1}$$

where $\bar{u}$ and $\bar{v}$ are averages of the velocity components in a given neighborhood and P and D are defined as

$$P = f_x \bar{u} + f_y \bar{v}, \tag{2}$$

$$D = \lambda^2 + f_x^2 + f_y^2. \tag{3}$$

This Gauss-Seidel iteration finds the values of u and v that minimize the function

$$E^2(x, y) = (f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2). \tag{4}$$

The values of u and v providing the minimum of Eq. 4 are the solution for the velocity vector problem (see the book, pp. 686–687).
The values of f_x, f_y, and f_t can be estimated from the image pair, for instance,

$$f_x(x, y) = f(x + 1, y, t) - f(x, y, t),$$

$$f_y(x, y) = f(x, y + 1, t) - f(x, y, t), \text{ and}$$

$$f_t(x, y) = f(x, y, t + 1) - f(x, y, t). \tag{5}$$

The coefficient λ is a Lagrange multiplier used in constrained nonlinear programming problems[2].
The Eqs. (15.9) in the textbook yield two equations that may be solved for λ^2 in Eq. 3:

$$\lambda^2 = \frac{f_x f_y v + f_x^2 u + f_x f_t}{\bar{u} - u}$$

$$= \frac{f_x f_y u + f_y^2 v + f_y f_t}{\bar{v} - v}. \tag{6}$$

In Eqs. 1, the averages $\bar{u}$ and $\bar{v}$ have a constant effect on the new values u^k and v^k. The effect of the image gradient vanishes when the gradient is perpendicular to the average velocity, i.e., $P = \nabla \boldsymbol{f} \cdot \bar{\boldsymbol{c}} = 0$. Otherwise, some fraction of the image gradient, $-\frac{P}{D}\nabla \boldsymbol{f}$, is added to the average velocity, see Fig. 2.
If the neighborhood were not taken into account, then optical flow would always be determined in the direction of the local image gradient. The method of Horn and Schunck makes a trade-off between velocity in the direction of the local image gradient at (x, y) and the average velocity in its neighborhood. The method is based on matching the direction of the image gradient to the average velocity (in P), and on the magnitude of the image gradient (in D), for instance.

[2]Minimization of a scalar function $F(\boldsymbol{x})$ subject to constraints $h_i(\boldsymbol{x}) \geq 0$ leads to minimization of the Lagrange function $L(\boldsymbol{x}, \boldsymbol{\lambda}) = F(\boldsymbol{x}) + \sum_{i=1}^{m} \lambda_i h_i(\boldsymbol{x})$.

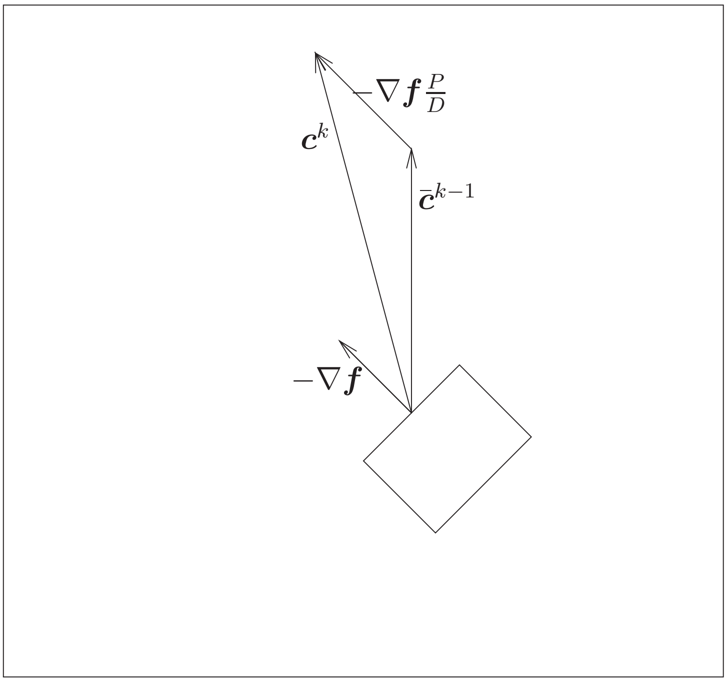

Figure 2: Local velocity is updated according to the average velocity, $\bar{c}^{k-1}$, in a given neighborhood, which is added to some fraction of the image gradient, $-\nabla f \frac{P}{D}$.

Notice The suggested initialization $\boldsymbol{c} = 0$ is not good because $\bar{\boldsymbol{c}}$ and P are then zero and any new velocity component values are also zero. A better initialization is provided in the textbook by Eq. (15.7), according to which the component of velocity $\boldsymbol{c}$ in the direction of image gradient $\nabla \boldsymbol{f}$ is $-f_t$ and therefore u and v may be initialized to

$$u^0 = \frac{-f_t f_x}{\sqrt{f_x^2 + f_y^2}},$$

$$v^0 = \frac{-f_t f_y}{\sqrt{f_x^2 + f_y^2}}. \tag{7}$$