T-61.5030 Advanced course in neural computing Karhunen, Peltonen

Exercise 8, Nov. 16, 2006

1. Use a simple network model to analyze how the amount of noise affects what kinds of representations should be used. Use the network model in Haykin's Problem 10.6. Make simplifying assumptions (Gaussian distribution, independence, etc.) if needed.

2. Consider a neuron that transforms a single input x into $y = g(x)$. Derive a learning rule for the neuron using the Infomax principle when

$$g(x) = \frac{1}{1 + exp(-(wx + w_0))} \ .$$

   Analyze the learning rule (what is its goal, other properties). What would happen if you generalized this principle to the multivariate case?

3. The n-step transition probability from state i to state j in a Markov chain is denoted by $p_{ij}^{(n)}$. Using the method of induction, show that

$$p_{ij}^{(1+n)} = \sum_k p_{ik} p_{kj}^{(n)} \ .$$

4. Calculate the steady-state probabilities of the Markov chain shown in Haykin, Fig. P11.4.

5. In this problem we consider the use of simulated annealing for solving the *traveling salesman problem* (TSP). You are given the following:

   - N cities
   - the distance between each pair of cities, d
   - a tour represented by a closed path visiting each city once and only once.

   The objective is to find a tour (i.e., permutation of the order in which the cities are visited) that is of minimal total length L. Here the different possible tours are configurations and the total length of a tour is the cost function to be minimized.

   (a) Devise an iterative method of generating valid configurations.

   (b) The total length of a tour is defined by

$$L_P = \sum_{i=1}^{N} d_{P(i)P(i+1)}$$

   where P denotes a permutation with $P(N+1) = P(1)$. Correspondingly, the partition function is

$$Z = \sum_P e^{-L_P/T}$$

   where T is a control parameter. Set up a simulated annealing algorithm for the TSP.