T-61.5030 Advanced course in neural computing Karhunen, Peltonen

Exercise 5, Oct. 19, 2006

1. Prove that PCA minimizes $E_{\mathbf{x},\mathbf{y}}[d(\mathbf{x},\mathbf{y})^2 - d(\mathbf{x}',\mathbf{y}')^2]$ where d is the Euclidean distance function, the $\mathbf{x}$ and $\mathbf{y}$ are original data samples, and $\mathbf{x}'$ and $\mathbf{y}'$ are data samples after PCA projection.

2. For the matched filter considered in Haykin, Example 8.2, the eigenvalue λ_1 and associated eigenvector $\mathbf{q}_1$ are defined by
$$\lambda_1 = 1 + \sigma^2$$
$$\mathbf{q}_1 = \mathbf{s}$$
Show that these parameters satisfy the basic relation
$$\mathbf{R}\mathbf{q}_1 = \lambda_1 \mathbf{q}_1$$
where $\mathbf{R}$ is the correlation matrix of the input vector $\mathbf{X}$.

3. Consider the maximum eigenfilter where the weight vector $\mathbf{w}(n)$ evolves in accordance with Haykin, Eq. (8.46). Show that the variance of the filter output approaches λ_{max} as n approaches infinity, where λ_{max} is the largest eigenvalue of the correlation matrix of the input vector.

4. Show that in Kernel PCA, the normalization of eigenvector $\tilde{\mathbf{q}}$ of the correlation matrix $\tilde{\mathbf{R}}$ is equivalent to the requirement that Haykin, Eq. (8.153) be satisfied.