Tik-61.246 Digital Signal Processing and Filtering

2. Mid Term Exam 13.12.1999 at 9-12. Halls A, C, D.

1. Are the following statements right or wrong? (A right answer gives +1 points, a wrong answer −1 points, no answer 0 points. The minimum is still 0 points and the maximum 3 points.)

   a) When converting an analog IIR type low pass filter to a digital filter using the bilinear transform, the order of the filter almost always increases.

   b) With the impulse-invariant method, there is a one-to one mapping from the frequency axis of the s-plane to the unit circle in the z-plane.

   c) If we use a Hamming window with length 25 to design a linear phase FIR filter, the group delay is always 12.

   d) By scaling a filter $H(z)$ with $\max\{|KH(z)|\} = 1$, we can prevent overflowing and improve the signal-to-noise ratio.

   (3p)

2. Suppose you want to design a first-order stable and causal high-pass filter having the following transfer function:

$$H(z) = K\frac{1 + bz^{-1}}{1 + az^{-1}} \ ,$$

   where the coefficients a, b, and K are real. The spesifications of the filter are as follows:

   1) the amplitude response is zero at zero frequency ($|H(e^{j0})| = 0$),
   2) the amplitude response is one at $\frac{1}{2}$ of the sampling frequency ($|H(e^{j\pi})| = 1$), and
   3) the attenuation at $\frac{1}{4}$ of the sampling frequency is 20 dB ($|H(e^{j\frac{\pi}{2}})| = 0.1$).

   Determine the coefficients of the filter, draw the zero-pole diagram, and sketch the amplitude response.

   (6p)

3. Consider two finite impulse response (FIR) systems having the following impulse responses:

$$\begin{aligned} h_1[n] &= \delta[n] + 2\delta[n-2] + \delta[n-4] \\ h_2[n] &= \delta[n] - \delta[n-4] \end{aligned}$$

   a) Determine the impulse response $h_c[n]$ and the transfer function $H_c(z)$ of the cascade connection of the above systems (scaling to unity is not required). Is the phase response linear? What is the value of the phase response at frequency $\omega = 0$? Explain the results or calculate them!

   b) Determine the impulse response $h_p[n]$ and the transfer function $H_p(z)$ of the parallel connection of the above systems (scaling to unity is not required). Is the phase response linear? What is the value of the phase response at frequency $\omega = 0$? Explain the results or calculate them!

   c) What is the unit step response of the cascade connection, i.e. the response of the system to the unit step sequence $\mu[n] = 1,\ n \geq 0,\ \mu[n] = 0,\ n < 0$. What is the unit step response like when n is large? Why?

   (6p)

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4. In the figure below, there is a flow chart illustrating a second-order digital IIR type filter. Draw all the possible locations of the poles (p_1, p_2) of the filter when the real coefficients a_1 and a_2 are quantized to three bits using sign-magnitude truncation. With the used precision we can thus represent the coefficient values $[-\frac{3}{4},\ -\frac{1}{2},\ -\frac{1}{4},\ 0,\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4}]$. Note that due to the real coefficients, the poles are complex conjugates: $p_1 = re^{j\theta}, p_2 = re^{-j\theta}$.

   How does the quantization of the coefficients affect the realization of the filter at different values of the frequency ω? Consider situations when a narrow-banded low-pass filter or a narrow-banded band-pass filter (with a passband at $\frac{1}{2}$ of the sampling frequency) is desired.

(6p)