# T-61.246 Digital Signal Processing and Filtering

2nd Mid Term Exam, December 12, 2001 at 9-12.
You are allowed to have a math reference book and a (graphical, programmable) calculator. You are not allowed to save notes in the calculator.

1. (3p) Are the following statements true or false? A right answer gives +0.5 points, no answer 0 points, and a wrong answer -0.5 points. The total point amount for this problem is, however, between 0–3 points.

   a) The feedback loop $G(z)$ in filter $H(z) = \frac{F(z)}{1-F(z)G(z)}$ may be delay-free if $F(z)$ contains delay registers.

   b) In the impulse-invariant method, the impulse response $h[n]$ is obtained by sampling the impulse response $h(t)$ of the corresponding analog filter.

   c) A weakness of the bilinear transform is that aliasing occurs if the original analog filter is not band-limited.

   d) The Gibbs phenomenon in the FIR window method can be removed by increasing the length of the rectangular window $w_{rec}[n]$, but at the same time frequency resolution is reduced.

   e) The numbers of computation steps (general complexities) in the FFT and DFT algorithms are $O(N\,log_2 N)$ and $O(N^2)$, respectively. Statement: When the length of the sequence to be transformed is $N = 1024 = 2^{10}$, the FFT is over 1000 times more effective than DFT (when computed with the above general complexities).

   f) With CD (compact disc) level sampling rate and 16-bit word length, the signal may contain 44100 different quantization levels.

   g) Truncating a discrete signal (input sequence) using e.g. the window $w_{rec}[n]$ causes distortion to the spectrum of the signal.

   h) By rounding a value to the nearest quantization level, the expected value of the quantization error is zero.

2. (3p) The amplitude response of a filter $H(z)$ is given below. The sampling rate of the system is 8 kHz.

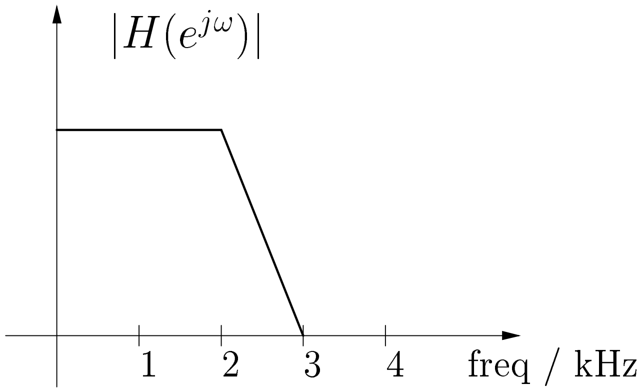


Figure 1: The amplitude response $|H(e^{j\omega})|$ of Problem 2.

   a) Increase the sampling rate by the factor $L = 3$. Draw the amplitude response of the obtained filter $H(z^3)$.

   b) What action should still be performed in order to obtain a lowpass filter from $H(z^3)$.

3. (6p) Consider the flow diagrams (a) - (f) of digital filters shown in the figure below. Which ones of the last five diagrams (b) - (f) have the same transfer function as the diagram (a). Note that some of the candidates can be quickly rejected with only a superficial inspection. Justify your answer on each candidate diagram.

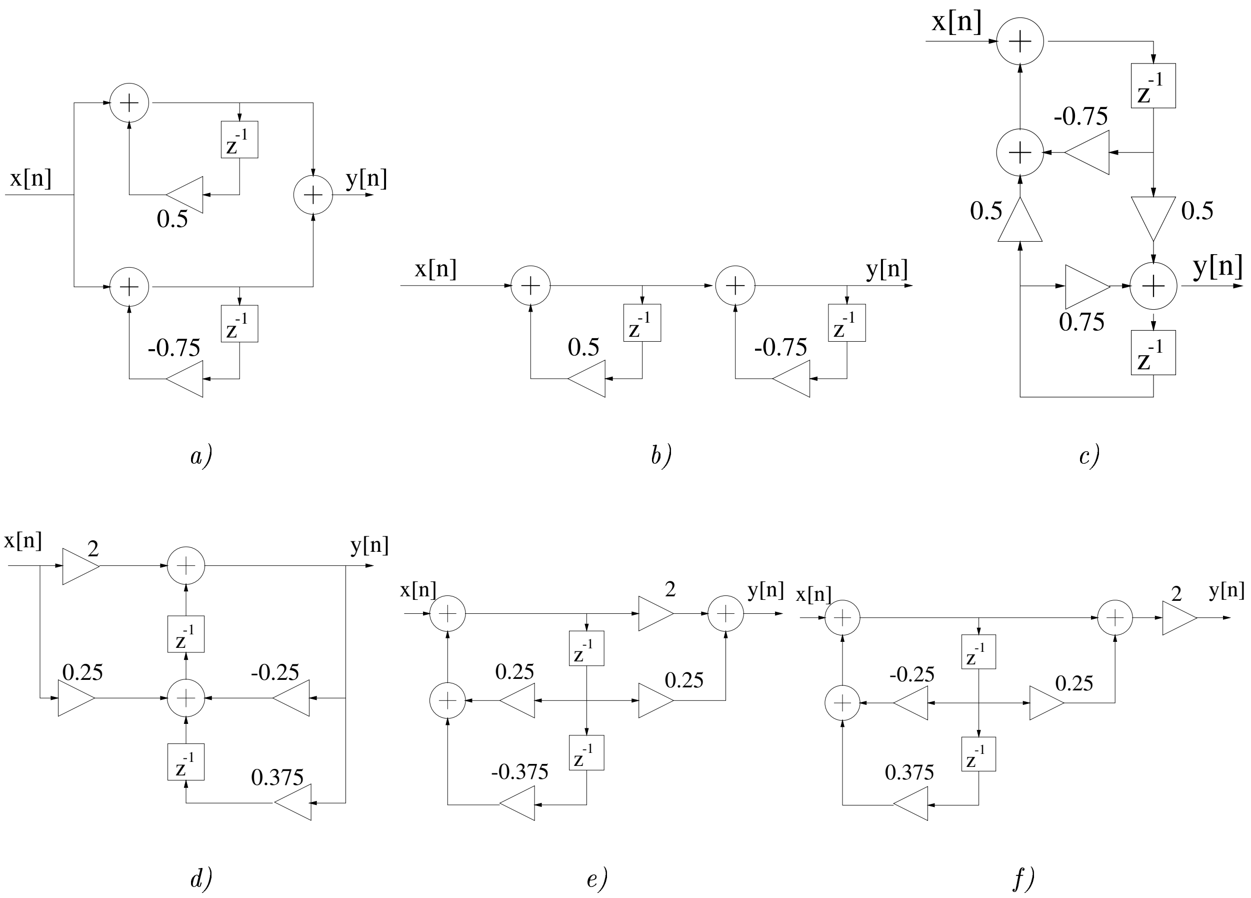


Figure 2: The flow diagrams of Problem 3.

4. (6p) Consider the following Butterworth-type lowpass filter

$$H_{LP}(s) = \frac{1}{s+1}$$

a) Form a first-order highpass filter with cutoff frequency $\Omega_c$ by substituting $H_{HP}(s) = H_{LP}(\frac{\Omega_c}{s})$

b) Implement a discrete first-order highpass filter $H(z)$, whose cutoff frequency (-3 dB) is $f_c = 3200$ Hz and sampling rate $f_s = 8000$ Hz, using the bilinear transform. Remember to prewarp the frequencies!

c) Draw the zero-pole diagram of the filter $H(z)$.

5. (6p) In the figure below, flow diagrams of two second-order IIR filters are displayed. Consider only the complex poles of the filters when the real-valued coefficients $a$ and $b$ are quantized to three bits using the sign-magnitude representation. The numbers possible to represent are thus $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$.

   Draw the possible locations of the complex poles of both systems and compare them. Notice that complex poles of a real-coefficient filter are complex conjugates ($p_1 = re^{j\theta}$, $p_2 = p_1^* = re^{-j\theta}$) and $1 + d_1 z^{-1} + d_2 z^{-2} = (1 - p_1 z^{-1})(1 - p_2 z^{-1})$.

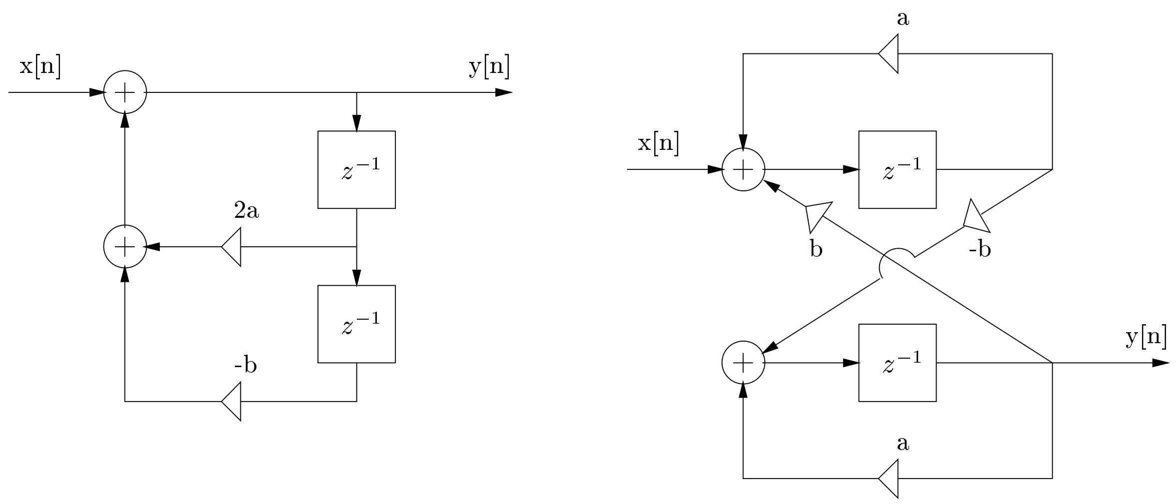


Figure 3: The flow diagrams of Problem 5.

**Some equations:**

$$e^{j\theta} = \cos(\theta) + j\ \sin(\theta)$$

$$y[n] = x[n] \circledast h[n] = h[n] \circledast x[n] = \textstyle\sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

$$X(z) = \textstyle\sum_{n=-\infty}^{+\infty} x[n]z^{-n}$$

$$X(e^{j\omega}) = \textstyle\sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$x[n] = \tfrac{1}{2\pi}\textstyle\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\mathrm{d}\omega$$

$$\textstyle\sum_{n=0}^{+\infty} a^n = \dfrac{1}{1-a} \qquad ,|a| < 1$$

$$H(e^{j\omega}) = |H(e^{j\omega})|e^{j\theta(\omega)}, \text{ where } \theta(\omega) = \arg\{H(e^{j\omega})\}$$

$$\tau(\omega) = -\frac{d\theta(\omega)}{d\ \omega}$$

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Z-transform:

$$X(z) = \textstyle\sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$x[n] \leftrightarrow X(z)$$

$$x[n-n_0] \leftrightarrow z^{-n_0}\, X(z)$$

$$a\, x[n] \leftrightarrow a\, X(z)$$

$$a^n\, \mu[n] \leftrightarrow 1/(1 - a\, z^{-1})$$

$$H(z) = Y(z)/X(z)$$

Bilinear transform:

$$s = (2/T)(1 - z^{-1})/(1 + z^{-1})$$

$$\Omega_c = (2/T)\ \tan(\omega_c T/2),$$

where $\Omega$ and $\omega$ are the angular frequencies of the analog and discrete filters, respectively