T-61.246 Digital Signal Processing and Filtering

2nd mid term exam 11th Dec 2002 at 9-12. Hall A.
You may use a (graphical) calculator. You must clear all extra memory in your calculator. There is an additional formulae table given in the exam.

1. (6p) Are the following statements true (T) or false (F)? A right answer gives +1 point, no answer 0 points, and a wrong answer –1 point. Reply to as many statements as you want; no explanations are needed. The total point amount for this problem is, however, between 0-6 points. If you want explicitely to comment on your choices, write down them separately.

   1) In the direct form the values of the coefficients of the transfer function can be directly (or negative) seen in the difference equation of the system.

   2) There cannot be any delay-free feedback loops in a realizable system.

   3) One possible polyphase realization for the FIR filter $H(z) = 1 + 0.3z^{-1} - 0.3z^{-2} - z^{-3}$ is $H(z) = E_0(z^2) + z^{-1}E_1(z^2)$, where $E_0(z) = 1 - 0.3z^{-1}$ and $E_1(z) = 0.3 - z^{-1}$.

   4) Digital FIR filter is computationally lighter compared to corresponding IIR filter because there is no recursive computation.

   5) A transfer function of a highpass filter is given by $H(z) = K(-0.0635 - 0.7289z^{-1} + 3.6692z^{-2} - 0.7289z^{-3} - 0.0635z^{-4})$. Statement: The coefficient K has to be $K = 0.2$, so that the maximum of the filter is scaled to unity (one).

   6) When using FIR window method, the impulse response of the ideal filter is convolved with the finite-length window function, and as a result an impulse response of a realizable filter is obtained.

   7) Sign bit $s = 1$ corresponds a negative number. Statement: The two's complement of a decimal number 0.125 using a sign bit and four bits is represented by $0_\Delta0010$.

   8) Sign bit $s = 1$ corresponds a negative number. Statement: The two's complement of a decimal number -0.625 using a sign bit and four bits is represented by $1_\Delta1010$.

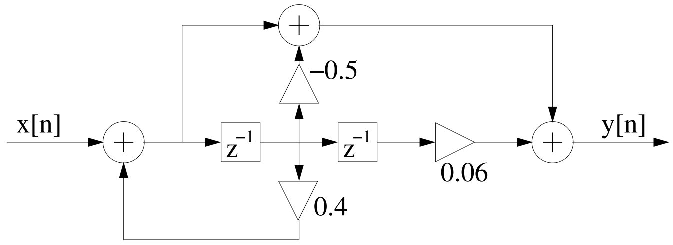
   9) The scaling of signal in order to supress the overflows in a filter also gives a better signal-to-noise ratio (SNR) in the filter.

   10) With an error-shaping structure the noise due to quantization can be removed from the system.

   11) The sampling frequency (multirate) is decreased by a factor $M = 3$, $f_s = (1/3)f_{s,old}$. Statement: The new sequence consists of only every third value from the original sequence.

   12) The sampling frequency (multirate) is increased to double $f_s = 2f_{s,old}$. Statement: There will be extra frequency components at frequencies $f_{image} = 2f_{old}$.

2. (6p) Consider a system depicted in the figure below.



   a) What is the transfer function of the filter $H(z)$?
   b) Is the system in the figure canonic?
   c) Let there be an input $x[n] = 0.2^n \mu[n]$. What is the output $y[n]$?

3. (6p) Consider an analog filter whose s-plane transfer function and impulse response are $H_a = 1/(s+1) \quad \leftrightarrow \quad h_a(t) = e^{-t}\mu(t)$.

   a) Determine the transfer function $H_I(z)$ of the digital filter corresponding the analog filter $H_a(s)$, when impulse-invariant method is used in the filter digitalization. In other words, take samples from the impulse response $h_a(t)$ of the analog filter at moments $t = nT_s$, where T_s is the sampling period.
   b) Determine the transfer function $H_B(z)$ of the digital filter corresponding the analog filter $H_a(s)$, when bilinear transform is used in the filter digitalization. It is assumed, that the frequency distortions have been taken into account in the design phase of the filter, and they do not need to be compensated. Use $s = (2/T_s)(1 - z^{-1})/(1 + z^{-1})$ for the transform.
   c) Normalize the sampling period to unity, i.e., $T_s = 1$. Compute the zeros and poles of $H_I(z)$ and $H_B(z)$, draw the pole-zero-diagrams, and sketch the amplitude responses of the filters. How do the two methods differ (in general)? Why?

4. (6p) The sampling frequency of a discrete-time signal $x[n]$ is to be lowered to 3/4 of the original sampling frequency f_s ($\omega_s = 2\pi$). The interesting band of the signal (bandlimited signal) is between $0 \ldots f_s/6$ (using normalized angular frequency $0 \ldots \pi/3$).

   Design the required multirate system and sketch the signal in the frequency domain after each component. Determine also the highest possible cut-off frequency of the aliasing suppression filter.

   a) Design the required multirate system. Which parts are needed and in which order. Sketch the diagram of the multirate system.
   b) Sketch the signal in the frequency domain after each component.
   c) Determine the highest possible cut-off frequency of the FIR aliasing suppression filter, so that the interesting band remains untouched.