T-61.246 Digital Signal Processing and Filtering

UPDATED. 2nd mid term exam / final exam 8th Dec 2003 at 16-19. Halls M, B, K.
If you are doing 2nd MTE, reply to problems 3, 4, 5, 6.
If you are doing final exam, reply to problems 1, 2, 3, 5, 6.
Write down, if you are doing 2nd MTE or final exam.
You may use a (graphical) calculator. You must clear all extra memory in your calculator. There is an additional formulae table given in the exam, but you can also use a math reference book of your own. **Write down all necessary steps which lead to the results.**

1. (6p, FINAL EXAM) Are the following statements true (T) or false (F)? A right answer gives +1p, no answer 0 p, and a wrong answer –0.5p. You do not need to explain. The total point amount for this problem is, however, between 0-6 points.

   a) The fundamental period of the sequence $x[n] = 2\cos(0.3\pi n + \pi/2)$ is $N_0 = 60$.
   b) The linear convolution of sequences $x[n] = \delta[n]+\delta[n-1]+\delta[n-2]$ and $h[n] = \delta[n]-\delta[n-2]+10\delta[n-5]$ is $y[n] = \delta[n] + \delta[n-1] - \delta[n-3] + 11\delta[n-4] - \delta[n-5]$.
   c) Consider a transfer function $H(z) = B(z)/A(z)$. The coefficients of the numerator polynomial $B(z)$ are called zeros and the coefficients of the denominator polynomial $A(z)$ are called poles.
   d) The phase response $\angle H(e^{j\omega})$ of a filter $H(z) = 0.5 - 0.5z^{-1}$ is linear.
   e) The bad thing in bilinear transform method is aliasing, if the analog filter is not band-limited.
   f) Using error-shaping structures the quantization noise can be modified and changed to a desired frequency band.

2. (6p, FINAL EXAM) Consider a filter with transfer function

$$H(z) = K \cdot \frac{1}{1 - 0.6z^{-1} + 0.9z^{-2}}$$

   a) Draw the pole-zero-diagram.
   b) Sketch the amplitude response $|H(e^{j\omega})|$. Is the filter lowpass / highpass / bandpass / bandstop?
   c) Explain briefly, why the filter is / is not stable. Does the coefficient K has effect on stability?
   d) What is the difference equation of the filter? You do not have to solve K.
   e) Compute the first values of the impulse response $h[n]$, when $n = 0\ldots3$. The filter registers are initialized to zero. A closed form is not needed, and you do not have to solve K.

3. (6p, MTE2, FINAL EXAM) Consider a signal

$$x(t) = 3\cos(2\pi f_1 t) + \cos(2\pi f_2 t) + 2\cos(2\pi f_3 t)\ ,$$

   where $f_1 = 4$ kHz, $f_2 = 16$ kHz and $f_3 = 20$ kHz.

   a) Sketch the spectrum $|X(j\omega)|$ of the signal $x(t)$ in range $-30\ldots30$ kHz.
   b) Determine the fundamental period T_0 of the signal $x(t)$.
   c) Apply the sampling frequency $f_s = 20$ kHz to the signal $x(t)$, and sketch the spectrum $|X(e^{j\omega})|$ of the sampled sequence $x[n]$ in range $0\ldots(f_s/2)$ kHz.
   d) Use an ideal lowpass filter (anti-aliasing)

$$H(j\omega) = \begin{cases} 1, & |f| < 9\text{kHz} \\ 0, & |f| \geq 9\text{kHz} \end{cases}$$

   and filter first $X_2(j\omega) = H(j\omega)X(j\omega)$. Sketch the spectrum $|X_2(j\omega)|$ in range $-30\ldots30$ kHz.

4. (6p, MTE2) Transform the filter structure in Figure 1 to a canonic (with respect to delay units) filter structure having the same transfer function $H(z)$. What is $H(z)$ and the order of that? Draw the block diagram again in canonic form using, e.g. Direct Form II.

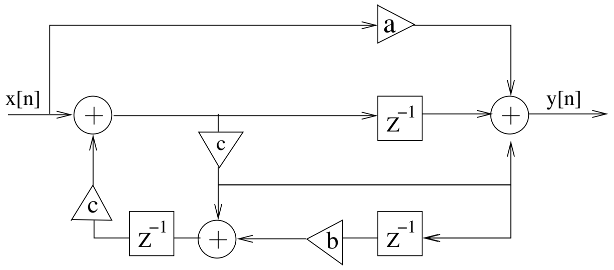

Figure 1: The filter in Problem 4.

5. (6p, MTE2, FINAL EXAM)

a) Draw the frequency response $H_{ideal}(e^{j\omega})$ of the ideal filter, when a lowpass filter with cut-off frequency $\omega_c = 2\pi/5$ is wanted.

b) Compute the impulse response $h_{ideal}[n]$ of the corresponding ideal filter. Write down the values for $n = -2 \ldots 2$. Hint: Inverse transform, after that you receive a non-causal infinite-length impulse response, which is a sinc function.

c) Compute the coefficients of FIR filter using window method (truncated Fourier series) and rectangular window of length 5 ($M = 2$): $w_s[n] = 1, \quad -M \leq n \leq M$. What is the order of FIR filter?

d) Do as in (c) but use a Hann window

$$w_h[n] = 0.5 \cdot \Big(1 + \cos\big(\frac{2\pi n}{2M}\big)\Big), \quad -M \leq n \leq M$$

6. (6p, MTE2, FINAL EXAM) Consider a multirate system in Figure 3. There are lowpass and highpass filters with cut-off frequency $\omega_c = \pi/2$, and upsamplers and downsamplers. Suppose that the power lost in downsampling is normalized in the output signal.

Which of the output $y_1 \ldots y_4$ in Figure 3 have to be summed together so that using the input spectrum $X(e^{j\omega})$ in Figure 2(a) the following output spectrum $Y(e^{j\omega})$ is processed.

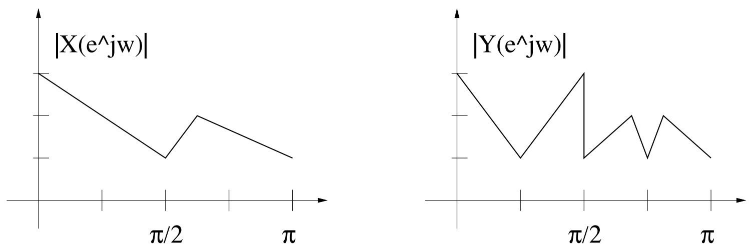

Figure 2: (a) Left, the input spectrum of Problem 6, (b) right, the output spectrum.

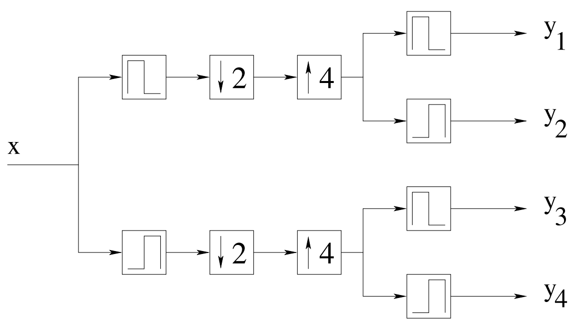

Figure 3: System in Problem 6, with ideal lowpass and highpass filters, and downsamplers ($M = 2$) and upsamplers ($L = 4$).