T-61.246 Digital Signal Processing and Filtering

2nd mid term exam / final exam 10th Dec 2003 at 9-12. Halls A, B, C.
If you are doing 2nd MTE, reply to problems 3, 4, 5, 6.
If you are doing final exam, reply to problems 1, 2, 3, 5, 6.
Write down, if you are doing 2nd MTE or final exam.
You may use a (graphical) calculator. You must clear all extra memory in your calculator. There is an additional formulae table given in the exam, but you can also use a math reference book of your own. **Write down all necessary steps which lead to the results.**

1. (6p, FINAL EXAM) Are the following statements true (T) or false (F)? A right answer gives +1p, no answer 0 p, and a wrong answer –0.5p. You do not need to explain. The total amount of points is 0-6p.

   a) Sequence $x[n] = -\cos((17\pi)n + \pi/6)$ is not periodic.
   b) The linear convolution of sequences $x[n] = \delta[n]+\delta[n-1]+\delta[n-2]$ and $h[n] = \delta[n]+\delta[n-1]+10\delta[n-2]$ is $y[n] = \delta[n] + 2\delta[n-1] + 12\delta[n-2] - 11\delta[n-3] - 1\delta[n-4]$.
   c) Consider a highpass filter (monotonic pass band) with transfer function $H(z) = K \cdot (1 - 0.8z^{-1})/(1 + 0.8z^{-1})$. Statement: The maximum of the filter is one, when $K = 1/9$.
   d) Using impulse-invariant method the digital impulse response $h[n]$ of the filter is sampled from the impulse response $h(t)$ of the analog filter.
   e) A digital filter is computed from the analog filter $H(s) = 10/(s + 10)$ using bilinear transform. Statement: Zero of the digital filter is at $z = -1$ and the pole is inside the unit circle.
   f) The numbers of computation steps (general complexities) in the FFT and DFT algorithms are $O(N\, log_2 N)$ and $O(N^2)$, respectively. Statement: When the length of the sequence to be transformed is $N = 1024 = 2^{10}$, the FFT is over 500 times more effective than DFT (when computed with the above general complexities).

2. (6p, FINAL EXAM) Four first-order filters $H_1(z) = 1/(1 - 0.7z^{-1})$, $H_2(z) = 1/(1 + 0.7z^{-1})$, $H_3(z) = 1/(1 - 0.8jz^{-1})$, and $H_4(z) = 1/(1 + 0.8jz^{-1})$ are in cascade and they form a filter

$$H(z) = K \cdot H_1(z) \cdot H_2(z) \cdot H_3(z) \cdot H_4(z)$$

   a) Draw the pole-zero-diagram of the filter $H(z)$.
   b) Sketch the amplitude response $|H(e^{j\omega})|$ so that the maximum is one.
   c) Determine the coefficient K so that the maximum of the amplitude response is one. Hint: Find the angular frequency ω_{max} and use $z = e^{j\omega_{max}}$ in equation $|H(z)| = 1$.
   d) What is the difference equation using input $x[n]$ and output $y[n]$ of the filter $H(z)$.

3. (6p, MTE2, FINAL EXAM) Consider a continuous-time real signal $x(t)$, whose spectrum $|X(j\omega)|$ is depicted in Figure 1. There are six components in positive frequencies (given in the figure). The angles of components are zero.

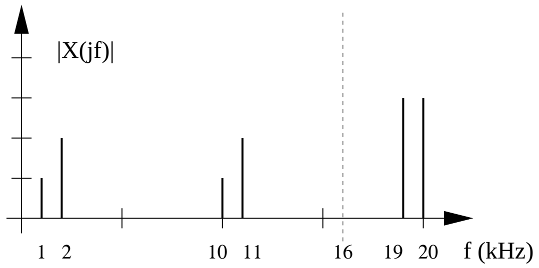

Figure 1: Spectrum $|X(j\omega)|$ in Problem 3. Components are at frequencies f_i: 1, 2, 10, 11, 19, 20 kHz.

   a) The signal $x(t)$ is periodic. What is the fundamental frequency f_0?
   b) Sample the signal with sampling frequency $f_s = 16$ kHz. Sketch the spectrum $|X(e^{j\omega})|$ of discrete-time sequence in range $0 \ldots f_s/2$.
   c) Use an ideal lowpass (anti-aliasing) filter

$$H(j\omega) = \begin{cases} 1, & |f| < 8\text{kHz} \\ 0, & |f| \geq 8\text{kHz} \end{cases}$$

   and filter $X_2(j\omega) = H(j\omega)X(j\omega)$. Sketch the spectrum $|X_2(j\omega)|$ in range $-30 \ldots 30$ kHz.

d) Sample the signal $x_2(t)$ using the same sampling frequency $f_s = 16$ kHz, after which the discrete-time sequence is returned back to analog $x_r(t)$ using ideal reconstruction. Write down $x_r(t)$ using sum of cosine functions $x_r(t) = \sum_i A_i \cos(2\pi f_i t)$.

4. (6p, MTE2) Consider the flow (block) diagram in Figure 2.

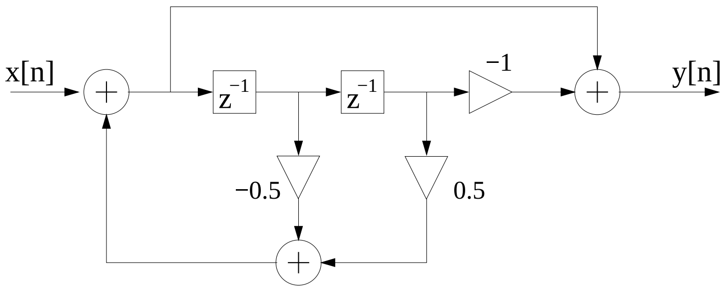

Figure 2: The flow diagram of Problem 4

a) What is the transfer function of the filter at its simpliest form?

b) Compute the poles and zeros. Draw the pole-zero-diagram. Is the structure in the figure canonic?

c) Compute the step response $s[n]$, that is, the response when the input is step function $\mu[n]$. What is $s[10]$? Which value $s[n]$ converges, as $n \to \infty$?

5. (6p, MTE2, FINAL EXAM)

a) Draw the frequency response $H_{ideal}(e^{j\omega})$ of the ideal filter, when a lowpass filter with cut-off frequency $\omega_c = 3\pi/5$ is wanted.

b) Compute the impulse response $h_{ideal}[n]$ of the corresponding ideal filter. Write down the values for $n = -2 \ldots 2$. Hint: Inverse transform, after that you receive a non-causal infinite-length impulse response, which is a sinc function.

c) Compute the coefficients of FIR filter using window method (truncated Fourier series) and rectangular window of length 5 ($M = 2$): $w_s[n] = 1, \quad -M \leq n \leq M$. What is the order of FIR filter?

d) Do as in (c) but use a Hann window

$$w_h[n] = 0.5 \cdot \left(1 + \cos\left(\frac{2\pi n}{2M}\right)\right), \quad -M \leq n \leq M$$

6. (6p, MTE2, FINAL EXAM) Consider a causal lowpass filter $H(z)$, whose passband ends at 4 kHz, stopband starts from 5 kHz and the sampling frequency is 12 kHz. The amplitude response is in Figure 3(a) and the start of the impulse response $h[n]$ in Figure 3(b). Modify the filter so that it can handle DAT-recordings with the sampling frequency of 48 kHz.

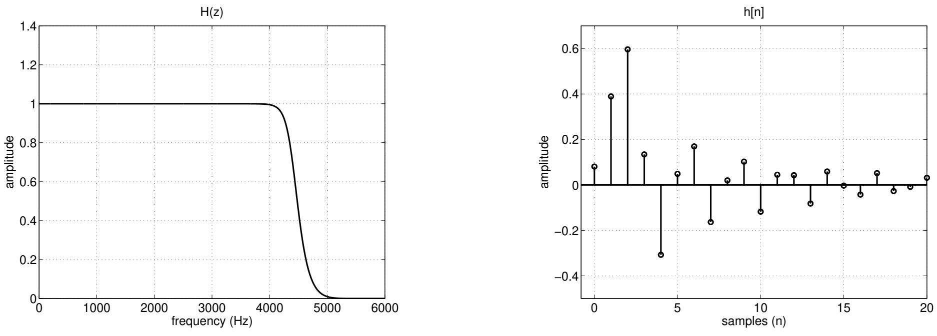

Figure 3: Problem 6: (a) left $|H(e^{j\omega})|$, (b) right $h[n]$.

a) Increase the sampling frequency with the factor $L = 4$. Draw the amplitude spectrum of the upsampled filtern $H(z^4)$ in range $0 \ldots 24$ kHz and the first ten values of the impulse response $h[n/4]$.

b) What has to be done so that the filter $H(z^4)$ works as a lowpass filter with the original cut-off frequencies?