Tik-61.246 Digital Signal Processing and Filtering

1st mid term exam 28th Oct 2000 at 10-13. Main building/CS-building.
You are allowed to have a mathematical reference book and (graphical) calculator. You are not allowed to save notes in calculator. Follow the instructions by assistant in the hall.

1. (2p) Examine if the following sequences are periodic. Calculate the basic period N of periodic sequences.
   a) $x_1[n] = 3\,cos(\frac{8}{31}n + \frac{1}{2}\pi)$
   b) $x_2[n] = \sum_{k=-\infty}^{\infty}(\delta[n-1-4k] - 2\delta[n+1-4k])$

2. (4p) Consider a linear, time-invariant, stable and causal discrete system, where the input $x[n]$ and output $y[n]$ are:

n	$x[n]$	$y[n]$
0	1	2
1	-2	1
2	0	?
3	1	?
4	2	?

   a) Define the impulse response $h[n]$ of the system using $x[n]$ and $y[n]$, and conditions that initial values of system are zero and it is form ($a,\ b,\ c$ and d constants):

$$h[n] = \begin{cases} a, & \text{when } n < 0 \\ b, & \text{when } n = 0 \\ c, & \text{when } n = 1 \\ d, & \text{when } n > 1 \end{cases}$$

   b) Calculate the missing values of $y[n]$.
   (Convolution $y[n] = h[n] * x[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$)

3. (6p) Consider a filter in Figure 1.

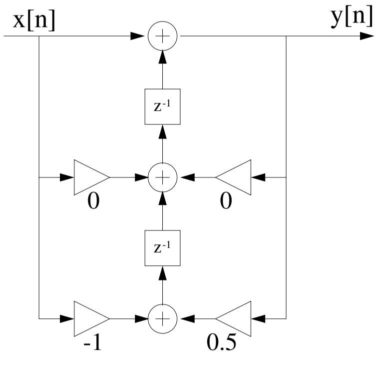
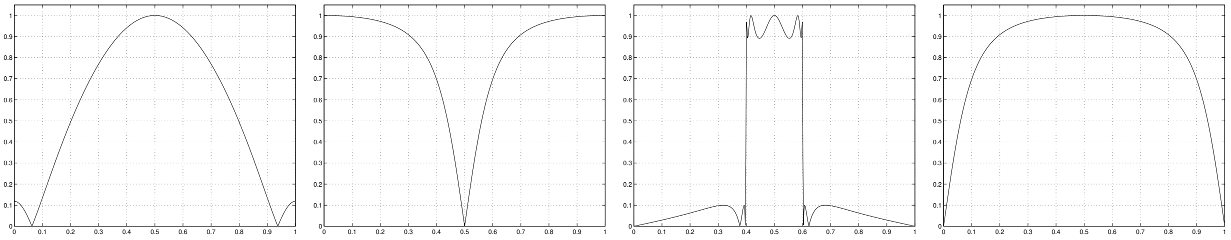


Figure 1: Block structure of Exercise 3

   a) Define the difference equation and transfer function $H(z)$ of the filter. (Hint: $ax[n-n_0] \leftrightarrow az^{-n_0}X(z)$, $H(z) = Y(z)/X(z)$)
   b) Calculate and draw the zero-pole-diagram of the filter.



i) *ii)* *iii)* *iv)*

Figure 2: Alternative amplitude responses of Exercise 3c

c) Choose the most appropriate amplitude response from Figure 2. In figures the frequency axis is $0..f_s/2$ (half of the sampling frequency) and amplitude axis is scaled in 0..1. What type of filter is it: FIR/IIR, low/high/bandpass/bandstop?

d) Define $H(z)$ as a combination of two blocks (not unique)

$$H(z) = H_1(z)H_2(z),$$

where $H_1(z)$ and $H_2(z)$ are first order transfer functions (one zero and one pole). Define the transfer functions $H_1(z)$ and $H_2(z)$ and sketch the amplitude repsonses for both.

4. (6p) Consider a signal

$$x(t) = 3\cos(2\pi f_1 t) + \cos(2\pi f_2 t) + 2\cos(2\pi f_3 t) \ ,$$

where $f_1 = 4$ kHz, $f_2 = 6$ kHz and $f_3 = 14$ kHz.

a) Define the basic period T of signal $x(t)$.

b) Sketch the magnitude spectrum $|X(j\omega)|$ of signal $x(t)$ in frequency range $-20\ldots20$ kHz.

c) Sample the signal $x(t)$ using sampling frequency $f_s = 10$ kHz and sketch the magnitude spectrum $|X(e^{j\omega})|$ of discrete sequence $x[n]$ in frequency range $-20\ldots20$ kHz.

d) Use ideal lowpass filter

$$H(j\omega) = \begin{cases} 1, & |f| < 9\text{kHz} \\ 0, & |f| \geq 9\text{kHz} \end{cases}$$

and filter $X_2(j\omega) = H(j\omega)X(j\omega)$. Sketch the magnitude spectrum $|X_2(j\omega)|$ in range $-20\ldots20$ kHz.

e) Sample the filtered signal $x_2(t)$ using sampling frequency $f_s = 10$ kHz and sketch the magnitude spectrum $|X_2(e^{j\omega})|$ of discrete sequence $x_2[n]$ in frequency range $-20\ldots20$ kHz.