T-61.246 Digital Signal Processing and Filtering

1st mid term exam, Sat 26.10.2002 at 10-13, main building.

It is not allowed to use any calculators or reference books. All concept papers should be returned.

A formulae table is delivered in the exam.

1. (6p) Let us examine a cascade LTI system in Figure 1 below. The impulse response $h_1[n] = \mu[n] - \mu[n-2]$. The total impulse response of the system $h[n]$ is given in the table below.

n	< 0	0	1	2	3	4	> 4
h[n]	0	1	0	0	4	3	0

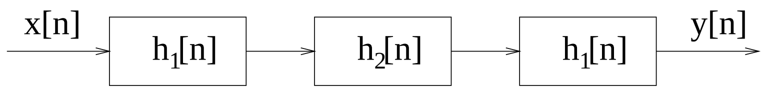

Figure 1: $h[n]$ consists of three LTI subsystems in cascade.

   a) Compute the impulse response $h_2[n]$.
   b) Compute the output of the system $h[n]$ for the input $x[n] = -2\delta[n+1] + \delta[n]$.

2. (6p) The difference equation of a LTI system is

$$y[n] = x[n] + x[n-1] - 2a\, y[n-1] - 2a^2\, y[n-2]$$

   where the coefficient a is real value, whose value will be set later.

   a) Draw the block diagram of the filter.
   b) Determine the transfer function of the filter (regions of convergence (ROC) need not to be mentioned here).
   c) Examine the frequence characteristics of the causal filter as a function of a using pole-zero-diagram, when $a \geq 0$.

3. (6p) Are the following statements true (T) or false (F)? A right answer gives +0.5 points, no answer 0 points, and a wrong answer –0.5 points. Reply to as many statements as you want; no explanations are needed. The total point amount for this problem is, however, between 0-6 points. Write down a similar table onto your answer paper as given below. If you want explicitely to comment on your choices, write down them separately.

1:		2:		3:		4:		5:		6:	
7:		8:		9:		10:		11:		12:	
13:		14:		15:		16:		17:		18:	
19:		20:		21:		22:		23:		24:	

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1) There does not exist a fundamental period T for a continuous-time signal $x(t) = \sin(\frac{6\pi}{11}t) + \sin(\frac{6}{11}t)$.

2) The fundamental period N for the sequence $x[n] = \cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n + \pi)$ is $N = 96$.

3) The sampling period used in CDs is 44.1 ms.

4) For discrete-time sequences $g_1[n] = \sin(0.8\pi n)$ and $g_1[n] = \sin(3.2\pi n)$ holds $g_1[n] = g_2[n]$ for each value n.

5) A discrete-time system $y[n] = \sum_{k=-3}^{3} x[n-k]$ is linear and stable.

6) A discrete-time system $y[n] = nx[n]$ is linear and time-invariant.

7) A discrete-time causal system does not predict future, so its impulse response is $h[n] = 0$, for all $n > 0$.

8) The frequencies of the signal components stay unchanged in LTI systems – only amplitudes and phases of components change.

9) Let $y[n] = x_1[n] \circledast x_2[n]$ and $v[n] = x_1[n - N_1] \circledast x_2[n - N_2]$.
Hence, $v[n] = y[n - (N_1 + N_2 - 1)]$.

10) The autocorrelation sequence has its maximum value, when the lag is zero.

11) With a linear and constant coefficient difference equation
$\sum_{k=0}^{N} d_k\, y[n-k] = \sum_{k=0}^{M} p_k\, x[n-k]$
can be depicted only impulse responses, whose length is $\max\{N, M\}$.

12) The inverse Fourier-transform of the frequency response is impulse response.

13) The order of the filter $y[n] + 0.3\,y[n-3] = x[n] - 0.6\,x[n-1] + 0.2\,x[n-2]$ is two.

14) The filter $H(e^{j\omega}) = \sum_{n=0}^{\infty}(-2\,e^{j\omega})^{-n}$ is a stable IIR.

15) In a transfer function of a feedback LTI system there is always a denominator polynomial of at least order of one.

16) Consider a second order LTI systems with poles $p_1 = 0.5$ and $p_2 = -0.4$ and zeros $z_1 = 0.5 + 0.4j$ and $z_2 = 0.5 - 0.4j$.
Statement: The transfer function $H(z)$ has real coefficients.

17) The transfer function $H(z) = 1/(1 + 5z^{-1} + 6z^{-2})$ can be defined to be stable but not causal at the same time by choosing a proper region of convergence.

18) If all poles of a LTI system are in the origin, then the filter is FIR.

19) The partial fraction expansion of the transfer function $H(z) = \frac{1}{1+0.2z^{-1}-0.35z^{-2}}$ is $H(z) = \frac{0.5}{1+0.7z^{-1}} + \frac{0.5}{1-0.5z^{-2}}$.

20) The transfer function $H(z) = 1 - 2z^{-2} + z^{-4}$ is a linear-phase filter.

21) The poles and zeros of a linear-phase filter have mirror-symmetry respect to the unit circle so that the poles of a stable filter are inside the unit circle.

22) A continuous-time signal $x(t) = \cos(2\pi f t)$, where the frequency $f = 98$ kHz, is sampled with the sampling frequency $f_s = 10$ kHz. There is a peak at 78 kHz in the discrete-time spectrum $|X(e^{j\omega})|$.

23) A continuous-time signal $x(t) = \cos(2\pi f t)$, where the frequency $f = 98$ kHz, is sampled with the sampling frequency $f_s = 10$ kHz. After that the signal is reconstructed with an ideal lowpass filter. The frequency to be heard is 8 kHz.

24) The sampling period should be at least two times big as the fundamental period of the highest signal component in order not to have aliasing in the sampling process.