T-61.246 Digital Signal Processing and Filtering

1st mid term exam, Mon 20.10.2003 at 16-19, hall A.

It is not allowed to use any calculators or reference books. All concept papers should be returned. A formulae table is delivered in the exam.

1) (2p) What are two angles $\omega_{1,2}$ in range $[-\pi \ldots \pi]$ in radians, for which $\cos(\omega) = -\sqrt{3}/2 \approx -0.866$. Hint: formulae table.

2) (4p) Are the following statements true (T) or false (F)? A right answer gives +0.5 points, no answer 0 points, and a wrong answer –0.5 points. Reply to as many statements as you want; no explanations are needed. The total point amount for this problem is, however, between 0-4 points. Write down a similar table onto your answer paper as given below. If you want explicitely to comment on your choices, write down them separately.

1:		2:		3:		4:		5:		6:	
7:		8:		9:		10:		11:		12:	

1) There exists a fundamental period T for a cont.-time signal $x(t) = \sin(\frac{3\pi}{17}\,t) + \sin(\frac{7\pi}{11}\,t + \pi/7)$.

2) The fundamental period N for the sequence $x[n] = \cos(\frac{2\pi}{6}\,n) + \sin(\frac{\pi}{4}\,n + \pi)$ is $N = 48$.

3) The sampling period (interval) used in CDs is 44100 Hz.

4) Causal discrete-time systems are always also LTI systems.

5) LTI filter with the impulse response $h[n] = (1/n)\,\mu[n]$, is stable.

6) Let $y[n] = x_1[n] \circledast x_2[n]$ and $v[n] = x_1[n+N_1] \circledast x_2[n+N_2]$.
Hence, $v[n] = y[n + (N_1 + N_2)]$.

7) The order of the filter $y[n] + 0.3\,y[n-2] = x[n] - 0.6\,x[n-1] + 0.2\,x[n-2]$ is two.

8) The amplification of an all-pass-filter is 1 for each frequency: $|H(e^{j\omega})| = 1$. Consider a second order LTI systems with poles $p_1 = 0.5$ and $p_2 = 0.8$ and zeros $z_1 = -0.5$ ja $z_2 = -0.8$. Statement: The filter is all-pass.

9) The partial fraction expansion of the transfer function $H(z) = \frac{1-0.5z^{-1}}{1-z^{-1}+0.24z^{-2}}$ is $H(z) = \frac{0.5}{1-0.6z^{-1}} - \frac{0.5}{1-0.4z^{-1}}$.

10) The transfer function $H(z) = 1 - z^{-1} - z^{-2} + z^{-3}$ is a linear-phase filter.

11) Two-point moving average filter is low-pass filter.

12) The Matlab command `subplot` can be used to plot the cosine signal into top axis in a window: `t = [0 : 1/100 : 1]; y = cos(2*pi*10*t); subplot(t, y, 1);`.

3) (6p) Let us examine a cascade LTI system in Figure 1 below. The following impulse responses are known:

$$
\begin{aligned}
h_1[n] &= \mu[n] - \mu[n-2] \\
h[n] &= \delta[n] - \delta[n-1] - 7\delta[n-2] - 7\delta[n-3] - 2\delta[n-4]
\end{aligned}
$$

a) (3p) Compute the output of the system $h[n]$ for the input $x[n] = -2\delta[n+1] + 2\delta[n]$.

b) (3p) Compute the impulse response $h_2[n]$. Is $h_2[n]$ causal?

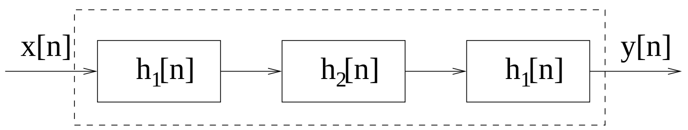

Figure 1: Problem 3, $h[n]$ consists of three LTI subsystems in cascade.

4) (6p) Consider the LTI filter shown in Figure 2.

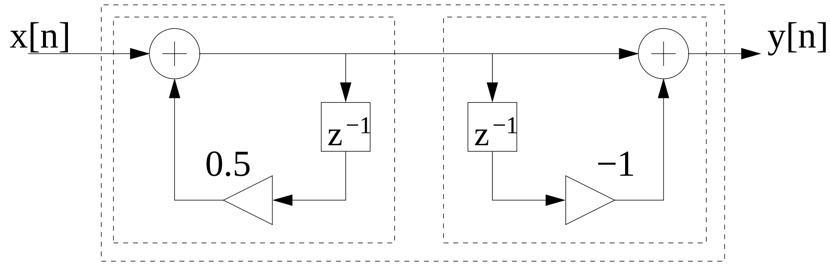

Figure 2: The block diagram of Problem 4

a) Construct the difference equation (between x and y) or a set of difference equations corresponding to the filter Hint: Use temporal variable w or apply cascade connection.

b) Define the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

c) Draw the zero-pole-diagram of the filter.

d) Based on the zero-pole-diagram, answer to the following questions: Of what type is the filter: lowpass / highpass / bandpass / bandstop / allpass? Is the filter stable?

e) What is the impulse response $h[n]$ in closed form?

5) (6p) It is possible to estimate the amplitude response of the filter from the pole-zero plot.

a) (4p) Connect pole-zero-plot to a corresponding amplitude response. There is one pole-zero-plot which does not fit to any amplitude response. Write down the three pairs (LETTER, number).

b) (2p) For which single figure (A-D, I-IV) does the following Matlab code refer to? How would you replace ???.

```
w = [0 : pi/128 : pi];
B = [1 0 0 0 0 0 0 0 -1];
A = [1 0 0.81];
zplane(B,???);
```

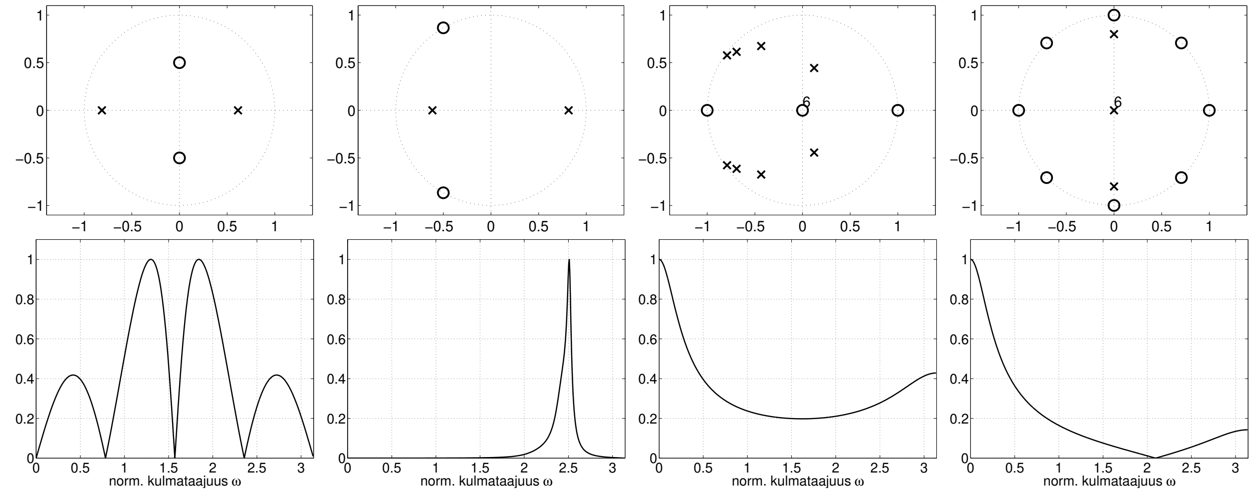

Figure 3: Problem 5, pole-zero-plots and amplitude responses. Top row from left to right A-D, bottom row from left to right I-IV.