T-61.246 Digital Signal Processing and Filtering

1st mid term exam, October 20, 2001 at 10-13.
You are allowed to have a math reference book and a (graphical, programmable) calculator. You are not allowed to save notes in the calculator. Follow the instructions of the assistants.

1. (2p) Are the following statements true or false? A right answer gives +0.5 points, no answer 0 points, and a wrong answer -0.5 points. The total point amount for this problem is, however, between 0–2 points.

   a) All causal and stable discrete-time systems are LTI systems.
   b) If the poles of a second-order filter $H(z)$ are at $z = \frac{1}{r}e^{\pm j\theta}$ and zeros at $z = re^{\pm j\theta}$, $r > 1$, the filter always has linear phase.
   c) If all the poles of a causal filter $H(z)$ are inside the unit circle, the filter is always also stable.
   d) An Nth order IIR filter always has N poles outside the origin.
   e) The length of the inpulse response of the filter in Figure 1 is infinite.
   f) A comb filter is an example of a FIR-type lowpass filter.

2. (4p) Sampling

   a) Draw the continuous-time signal $x(t) = \cos(2\pi f_1 t + \theta)$ in the range $t \in [0, 0.05]$, when $f_1 = 60$Hz and $\theta = \pi$.
   b) The signal $x(t)$ is sampled with sampling frequency $f_s = 50$Hz. Draw the absolute value $|X(e^{j\omega})|$ of the spectrum of the sampled signal $x[n]$ in the range $[0, f_s/2]$.
   c) Draw the continuous-time signal $\hat{x}(t)$ reconstructed from the base band $[-f_s/2, f_s/2]$ in the range $t \in [0, 0.05]$.

3. (6p) Consider the filter shown in Figure 1.

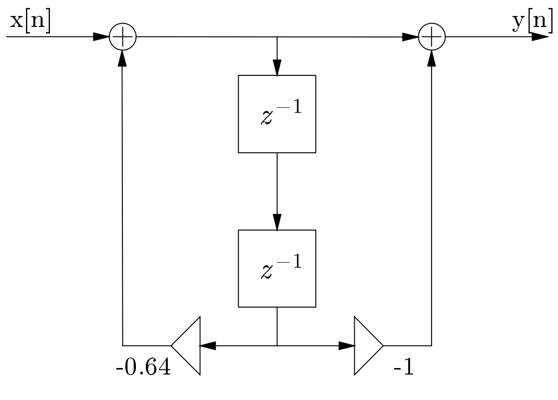

Figure 1: The filter in Problem 3

   a) Construct the difference equation or a set of difference equations corresponding to the filter.
   b) Define the transfer function $H(z) = \frac{Y(z)}{X(z)}$. z-transform: $ax[n - n_0] \leftrightarrow az^{-n_0}X(z)$
   c) Draw the zero-pole-diagram of the filter.
   d) Based on the zero-pole-diagram, answer to the following questions: Of what type is the filter: lowpass / highpass / bandpass / bandstop / allpass? Is the filter stable?

TURN THE PAPER!

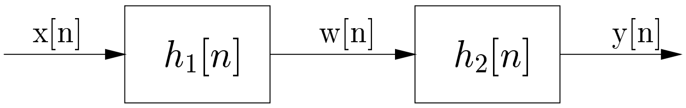

Figure 2: The cascade connection of Problem 4

4. (6p) Consider a cascade connection $h_c[n]$ of two linear, time-invariant, and causal discrete-time systems $h_1[n]$ and $h_2[n]$ as shown in Figure 2. The input-output relation of the system is given in the table below.

n	$x[n]$	$w[n]$	$y[n]$
0	1	1	1
1	-1	-2	-5
2	0	2	9
3	0	-1	-9
4	0	0	5
5	0	0	-1
6	0	0	0
7	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$

a) Both systems $h_1[n]$ and $h_2[n]$ are of FIR type. Determine the lengths of the inpulse responses (the part that is nonzero).

b) Define the inpulse response $h_c[n]$ of the cascade.

c) Draw the inpulse response $h_2[n]$ of the latter system in the cascade.

d) Define the transfer function $H_c(z)$ of the cascade. Solve the poles and zeros (hint: you can solve them separately for $H_1(z)$ and $H_2(z)$). Is the filter specified by $H_c(z)$ a lowpass, a highpass, or a bandpass filter.

Hints: The following properties hold for cascade connections:

$$h_c[n] = h_1[n] \circledast h_2[n]$$

$$H_c(z) = H_1(z)H_2(z)$$

Other equations:

$$e^{j\theta} = \cos(\theta) + j\ \sin(\theta)$$

$$y[n] = x[n] \circledast h[n] = h[n] \circledast x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\mathrm{d}\omega$$

$$\sum_{n=0}^{+\infty} a^n = \frac{1}{1-a} \quad , |a| < 1$$