Tik-61.246 Digital Signal Processing and Filtering

1. Mid Term Exam 8.11.1999 at 12-15. Halls C, D, G.

n	$\delta[n]$	$h[n]$
0	1	1
1	0	1
2	0	0
3	0	0
4	0	0

1. a) Suppose that we have a system, whose input-output-relation is $y[n] = x[n] + nx[n-1]$. When feeding in $\delta[n]$ the system gives the output $h[n]$, see the table. Is the system LTI (linear and time/shiftinvariant). Explain. (1p) Is the system stable? Explain. (1p)

   b) Suppose that we have a FIR filter whose transfer function is $H(z) = 1 - z^{-8}$. Calculate all zeros of $H(z)$. (1p)

2. a) There is a block diagram in the figure 1 below. Derive the corresponding difference equation using x and y. (1p)

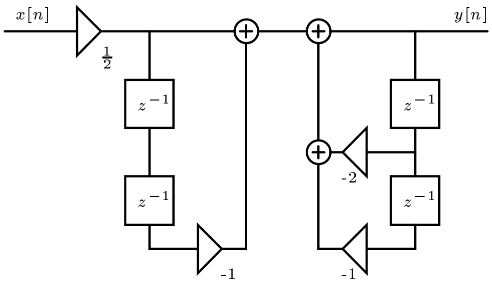


Figure 1: A block diagram for Problem 2.

   b) z-transform the difference equation and show the transfer function $H(z)$ in its simplest form. (1p)

   c) Draw the pole-zero-diagram and answer whether the system is stable. (1p)

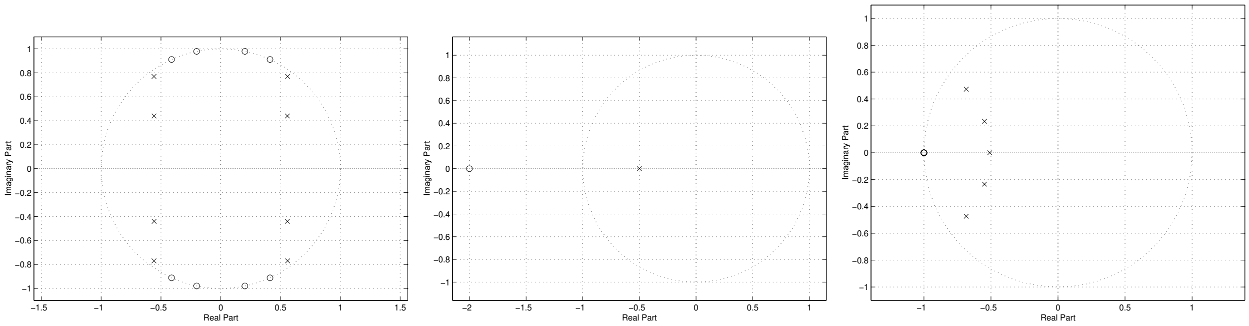
3. There are three pole-zero diagrams of three LTI systems in the figure 2. Answer for each subfigure i, ii, iii the following questions. (3×1p)

   a) Draw the amplitude response of the system. Scale the maximum amplification to 1 (0 dB) and normalize the frequency axis to $0..\pi$ (half of the unit circle).

   b) Is the filter FIR or IIR?

   c) What is its order?

   d) Is the system lowpass, highpass, bandpass, bandstop or allpass filter?



i) *ii)* *iii)*

Figure 2: Pole-zero diagrams for Problem 3

4. The transfer function of a LTI-system is

$$H(z) = \frac{1 + z^{-2}}{1 - 0.6z^{-1} - 0.72z^{-2}} \ .$$

   a) Calculate the poles and zeros, and draw a pole-zero diagram. (1p)
   b) What is the region of convergence (ROC) for the causal system? (1p)
   c) What is the region of convergence (ROC) for the stable system? (1p)
   d) Calculate the impulse response $h[n]$ for the causal system. (2p)
   e) For a stable LTI-system $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$. Is the causal system from question 4d also stable? Explain with the $h[n]$. (1p)

5. We have a signal

$$x_1(t) = \cos(2\pi f_1 t) + 2\cos(2\pi f_2 t) + 5\cos(2\pi f_3 t) \ ,$$

where $f_1 = 2$ kHz, $f_2 = 3$ kHz and $f_3 = 5$ kHz.

   a) Is the signal $x_1(t)$ periodic? If so, what is the basic period length T? (1p)
   b) Draw the absolute value of spectrum (magnitude spectrum) $|X_1(j\omega)|$ of signal $x_1(t)$ in frequency axis $-10 \ldots 10$ kHz. (1p)
   c) Use an ideal lowpass filter

$$H(j\omega) = \begin{cases} 1, & f < 4\text{kHz} \\ 0, & f \geq 4\text{kHz} \end{cases}$$

   and filter $X_2(j\omega) = H(j\omega)X_1(j\omega)$. Draw the absolute value of spectrum $|X_2(j\omega)|$ in frequency axis $-10 \ldots 10$ kHz. (1p)
   d) Sample the filtered signal $x_2(t)$ with sampling frequency of $f_T = 5$ kHz and draw the absolute value of spectrum $|X_2(e^{j\omega})|$ of the sequence $x_2[n]$ in frequency axis $-10 \ldots 10$ kHz. (2p)
   e) Reconstruct a continuous signal $x_3(t)$ from the discrete sequence $x_2[n]$ and draw it in time axis $0 < t < 0.5$ ms. What is the maximum value of $x_3(t)$? (1p)