Tik-61.246 Digital Signal Processing and Filtering

Exam/Mid Term Exam, Mon 20.12.1999 at 9-12. Halls T1 and T2.

WRITE DOWN, IF YOU DO 1st OR 2nd MID TERM EXAM, OR THE EXAM Tik-61.246 (this autumn) / Tik-61.146 (the old course)
1st Mid Term Exam: problems 1, 2, 3 and 4
2nd Mid Term Exam: problems 5, 6, 7 and 8
Exam: problems 2, 4, 6, 7 and 8

n	$\delta[n]$	$h[n]$
0	1	2
1	0	1
2	0	0
3	0	0

1. a) Consider a system, whose input-output-relation is $y[n] = x[n+1] + 2x[n] + x[n-1]$. Some output values $h[n]$ with the input $\delta[n]$ are calculated in the table beside. Is the system LTI (linear and time/shiftinvariant)? Explain. Is it causal? Explain.

   b) Consider the frequence response of a FIR filter $H(e^{j\omega}) = e^{-j2\omega} + 2e^{-j\omega} + 2e^{j\omega} - e^{j2\omega}$. Use Euler's formula $e^{j\omega} = \cos\omega + i\sin\omega$ and represent the frequence repsonse in a simple form with sines and cosines.

(3p)

2. a) Show that the z-transform of a convolution of two sequences is the product of z-transforms of these sequences,

$$Z\{x[n] * y[n]\} = X(z)Y(z)$$

   Convolution: $x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$. z-transform: $Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$.

   b) Inverse transform the transfer function $H(z) = \frac{2z^{-1}}{1-0.5z^{-1}}$ in order to get the impulse repsonse in time-domain.

(6p)

3. There are pole-zero-diagrams of three LTI-systems in the figure 1. Answer for each diagram i, ii, iii the following questions.

   a) Sketch the amplitude response of the system, where maximum amplification is scaled to 1 (0 dB) and the frequency is normalized in $0..\pi$ (half of unit circle).

   b) Is the filter FIR or IIR?

   c) What is the order of the filter?

   d) Is the filter lowpass, highpass, bandpass, bandstop or allpass?

(6p)

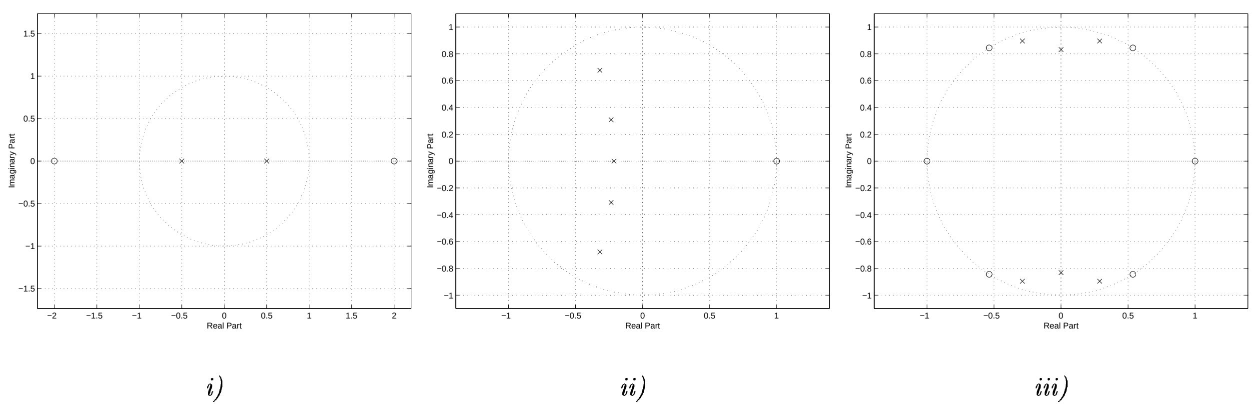

Figure 1: Pole-zero-diagrams of the problem 3

4. Consider a continuous signal

$$x_1(t) = \cos(2\pi f_1 t) + 2\cos(2\pi f_2 t) + 4\cos(2\pi f_3 t) \ ,$$

where $f_1 = 3$ kHz, $f_2 = 5$ kHz and $f_3 = 6$ kHz.

a) What is the basic period T of the periodic signal $x_1(t)$?

b) Sketch the magnitude spectrum $|X_1(j\omega)|$ of the signal $x_1(t)$ in frequencies $-8 \ldots 8$ kHz.

c) The signal is filtered with an anti-aliasing, analog filter before sampling. The amplitude response of the anti-aliasing filter is

$$|H(j\omega)| = \begin{cases} 1, & |f| < 4 \text{ kHz} \\ 0.1, & |f| \geq 5 \text{ kHz} \end{cases} \ .$$

Sample the filtered signal $x_2(t)$ with sampling frequency $f_T = 8$ kHz and draw the magnitude spectrum $|X_3(e^{j\omega})|$ of the sequence $x_3[n]$ in frequencies $-8 \ldots 8$ kHz.

(6p)

5. Are the following statements false or true? (A correct answer: +1p, a wrong answer: −1p, no answer: 0p. The minimum points of the problem is 0p and maximum 3p.)

a) In the inverse bilinear transform the unit circle in the z-domain maps one-to-one into the frequency axis of s-domain.

b) The discrete Fourier-transform calculated with FFT-algoritm is not so accurate as calculated by definition (DFT), but with N big enough the difference is not significant.

c) The phase response of a FIR filter can be non-linear, which implies that the group delay $\tau(\omega) = -d\theta(\omega)/d\omega$ is not constant.

d) In the FIR filter design, the length of the windowing function defines always also the length of the filter.

(3p)

6. Consider a FIR filter, whose transfer function is

$$H(z) = 1 + 2.5z^{-1} + z^{-2} \ .$$

a) Draw the pole-zero-diagram of the filter and sketch the amplitude response of the filter. What type of filter is it?

b) What is the phase response of the filter? Explain with use of impulse response or calculate!

c) Replace all delay registers by four-time delays ($z^{-1} \rightarrow z^{-4}$). Draw now the pole-zero-diagram of the filter and sketch the amplitude response of the filter. What type of filter is it now?

(6p)

7. Consider an analog transfer function $H_a(s) = (s+a)/[(s+a)^2 + b^2]$, where coefficients a and b are real. The pole-zero-diagram (in s-domain) and amplitude response of the filter are shown in figure 2.

Notice! You don't have to calculate any transfer functions in z-domain. Sketching the asked figures is enough.

a) Sketch the amplitude response and pole-zero-diagram of a digital filter designed with the impulse-invariant method.

b) Sketch the amplitude response and pole-zero-diagram of a digital filter designed with the bilinear transform.

c) Explain briefly, how do methods in a) and b) differ?

(6p)

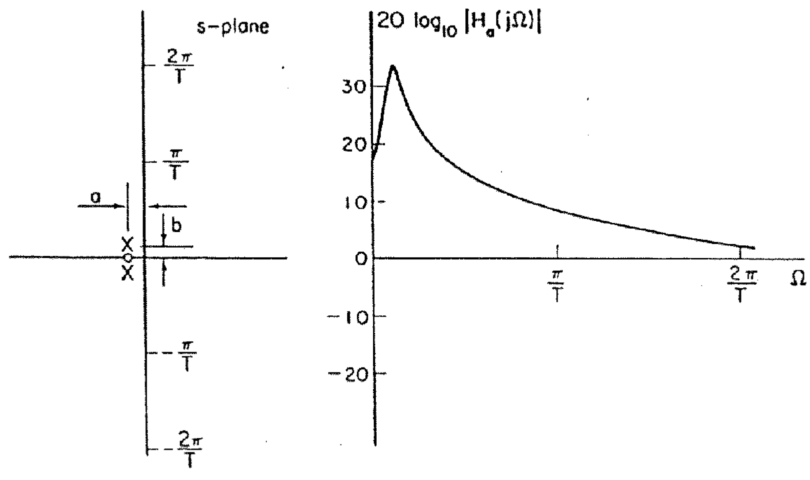

Figure 2: The pole-zero-diagram (left) and amplitude response (right) in s-domain

8. Quantization error can be compensated with error feedback. Without compensation $e[n]$ is a pure quantization error, in other words, $e[n] = y[n] - x[n]$. In the compensated structure the error sequence $e[n]$ is the difference between the output $y[n]$ and the compensated input signal $w[n]$, but the total error e_{tot} is still the difference between the output and input, $e_{tot} = y[n] - x[n]$. In the figure 3 there is an error feedback system of the second order. How does the spectrum of noise change as a function of b, when $b \in [-2, 2]$? What happens with the variance of the error? Assume that the original noice spectrum is uniform (white noise).

(6p)

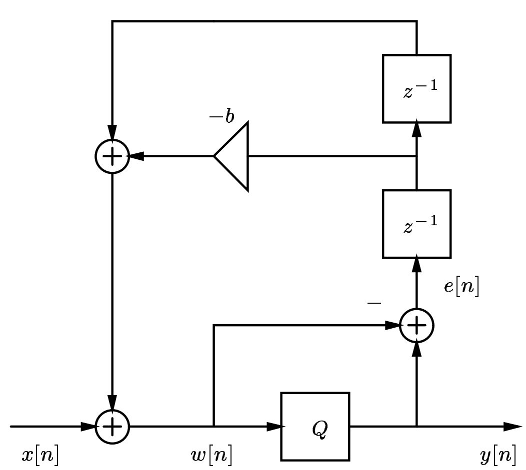

Figure 3: The system of the problem 8.